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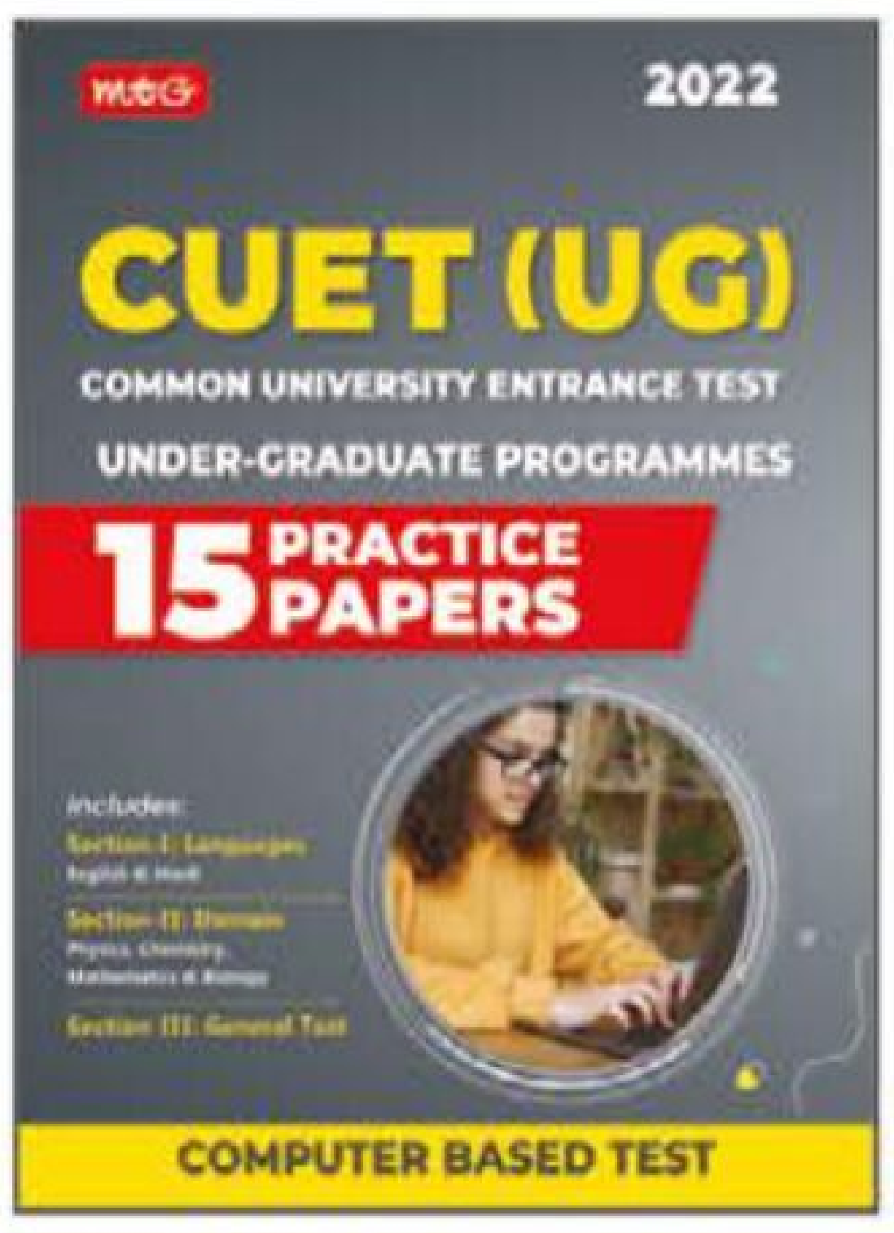

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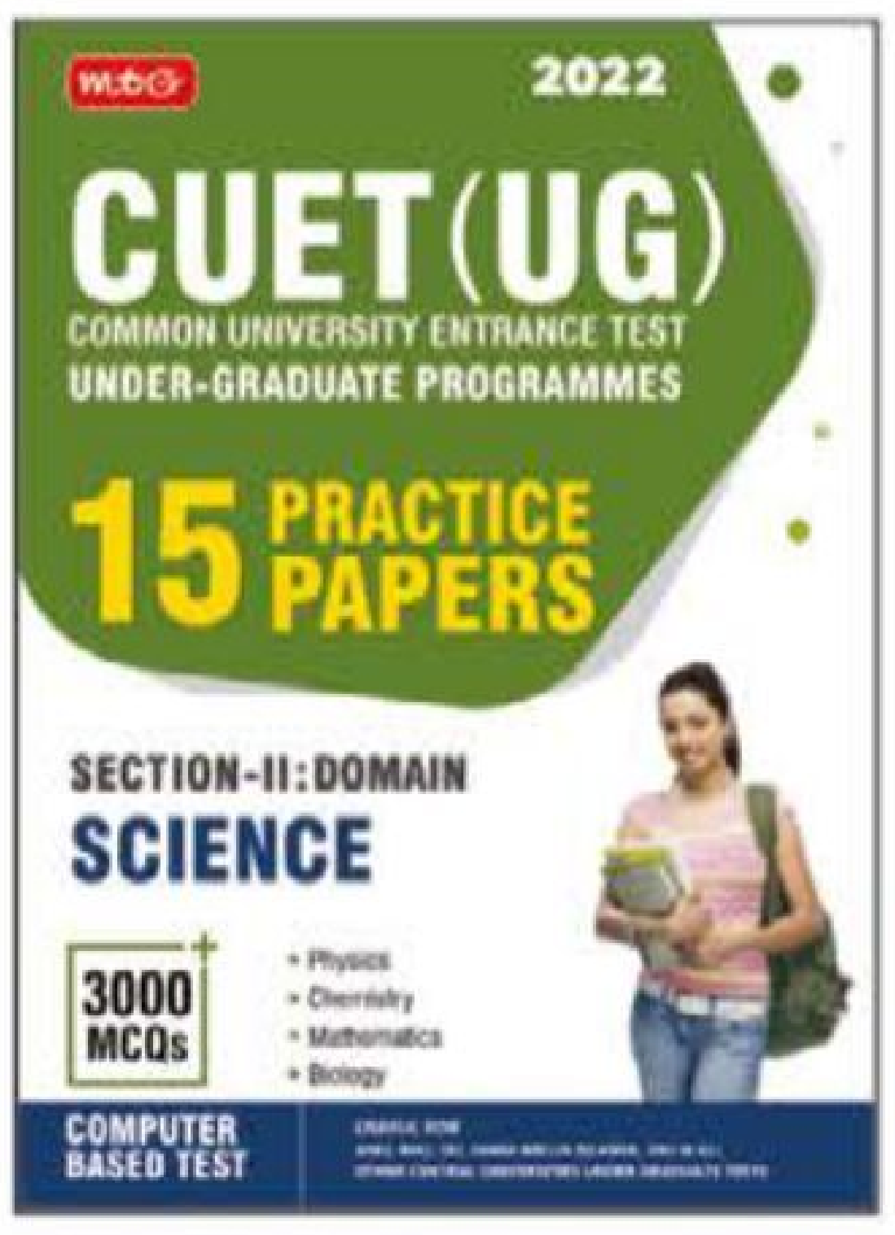

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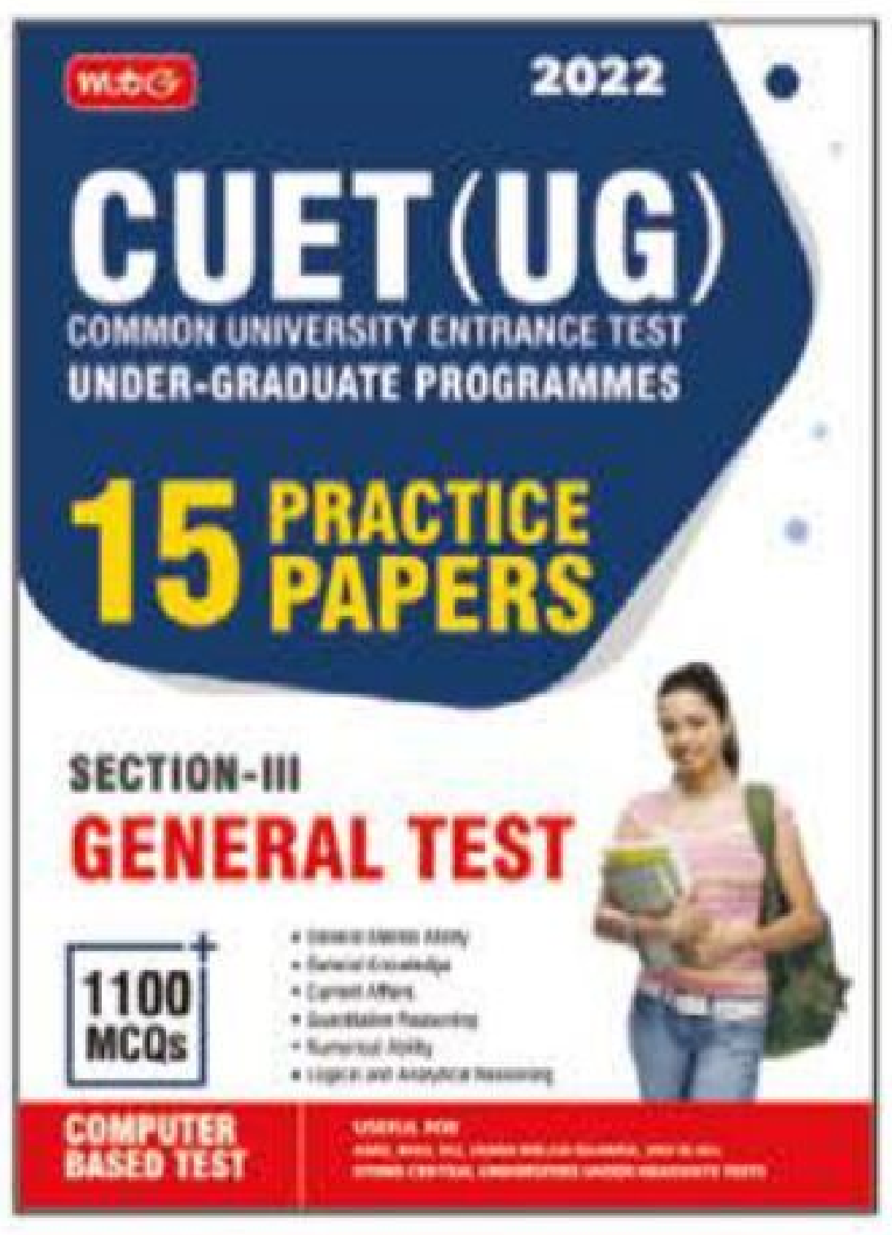

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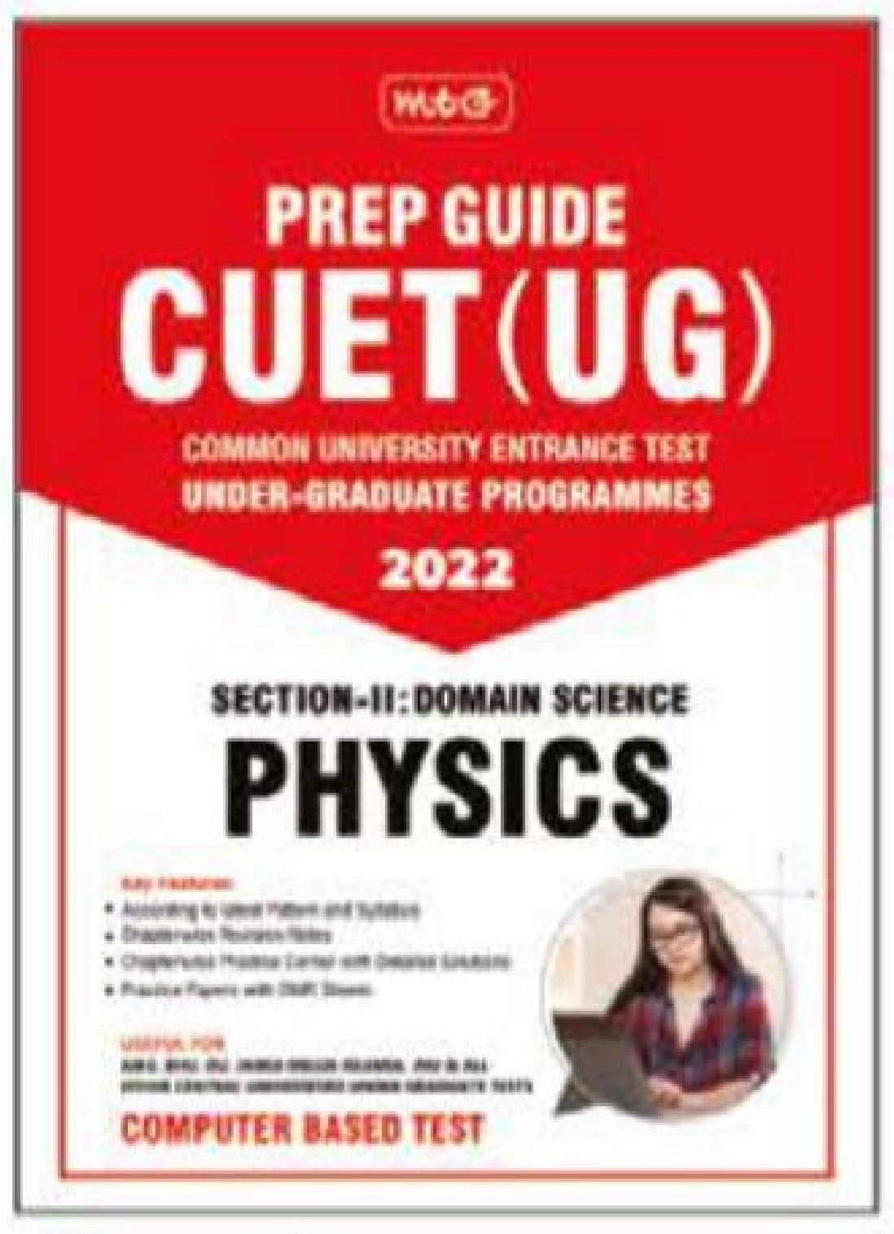

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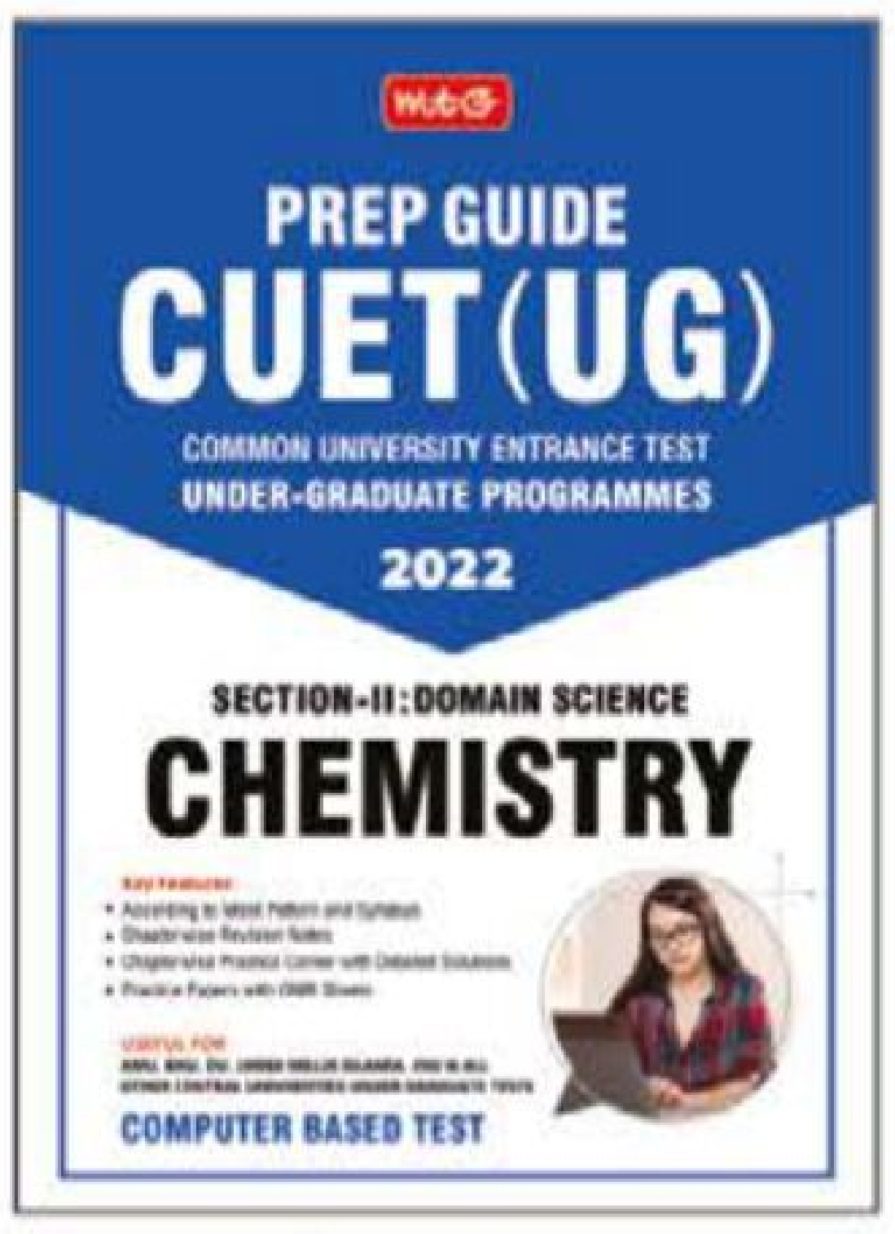

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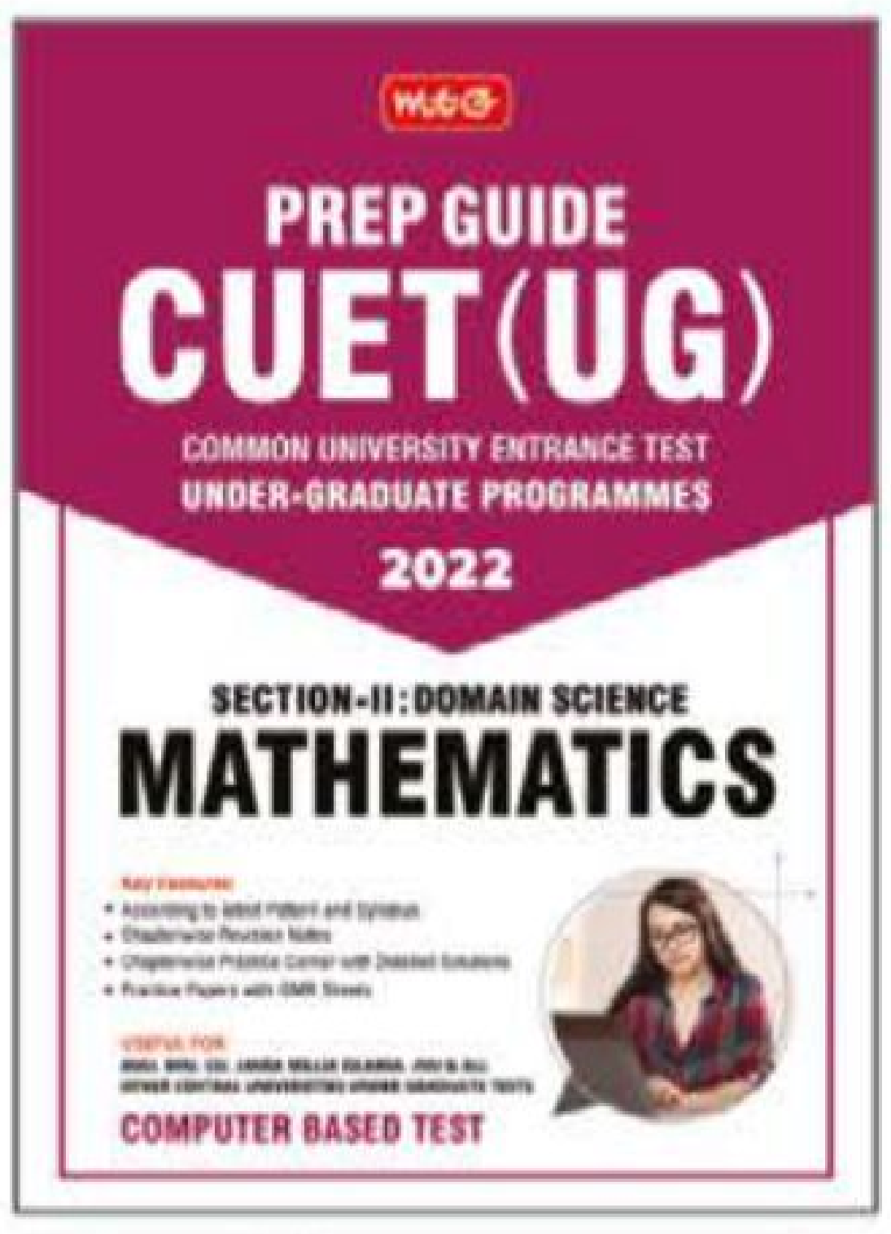

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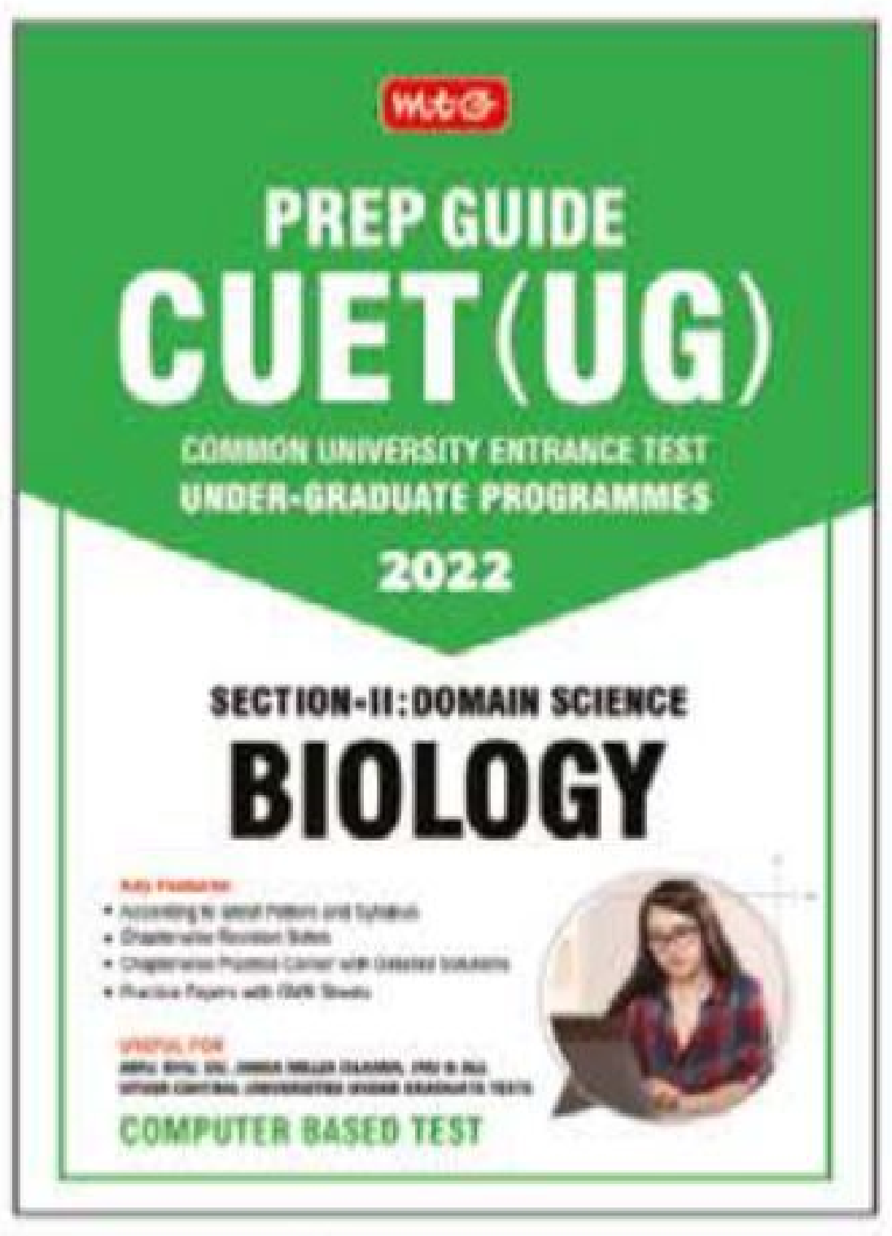

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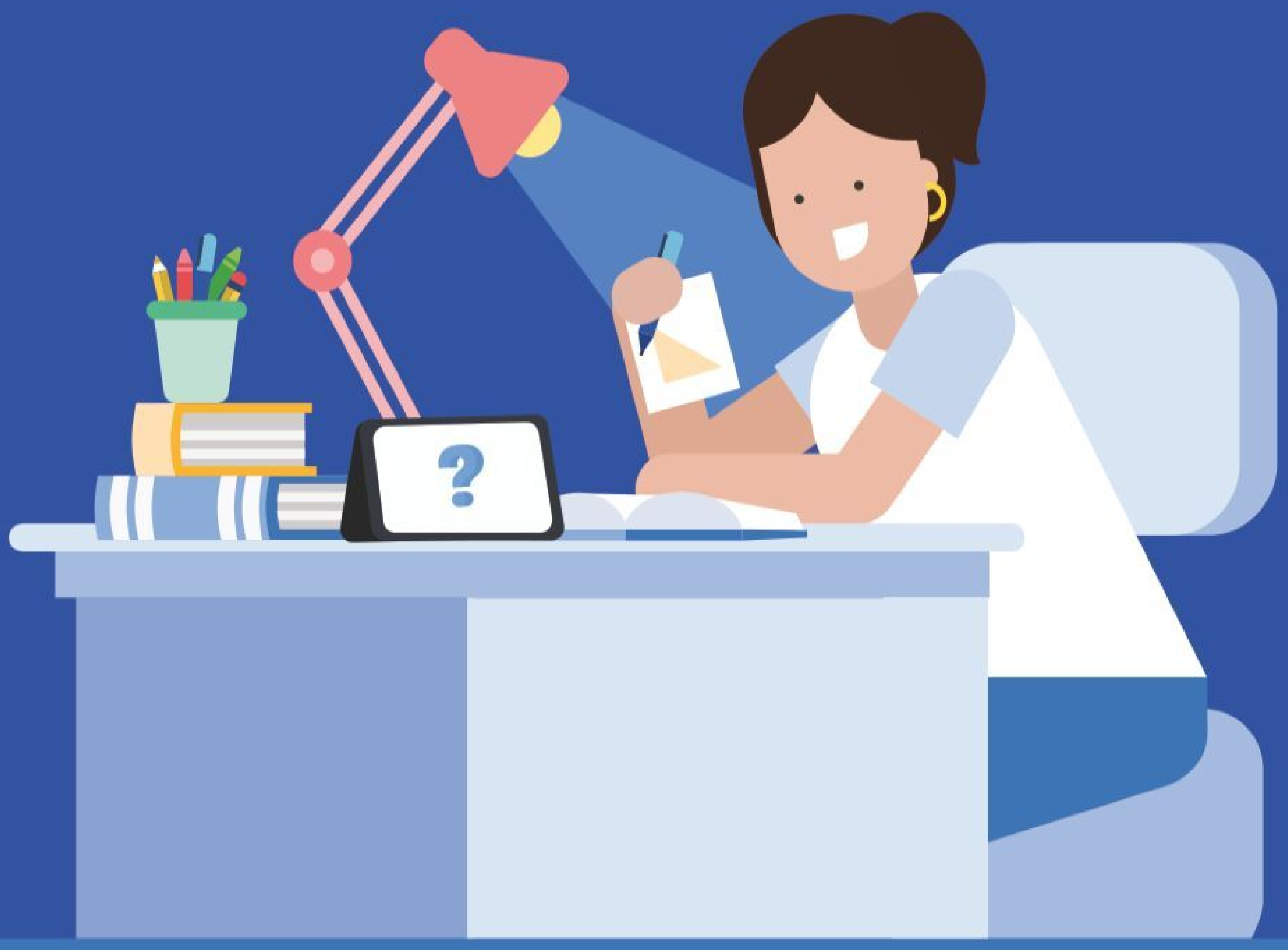

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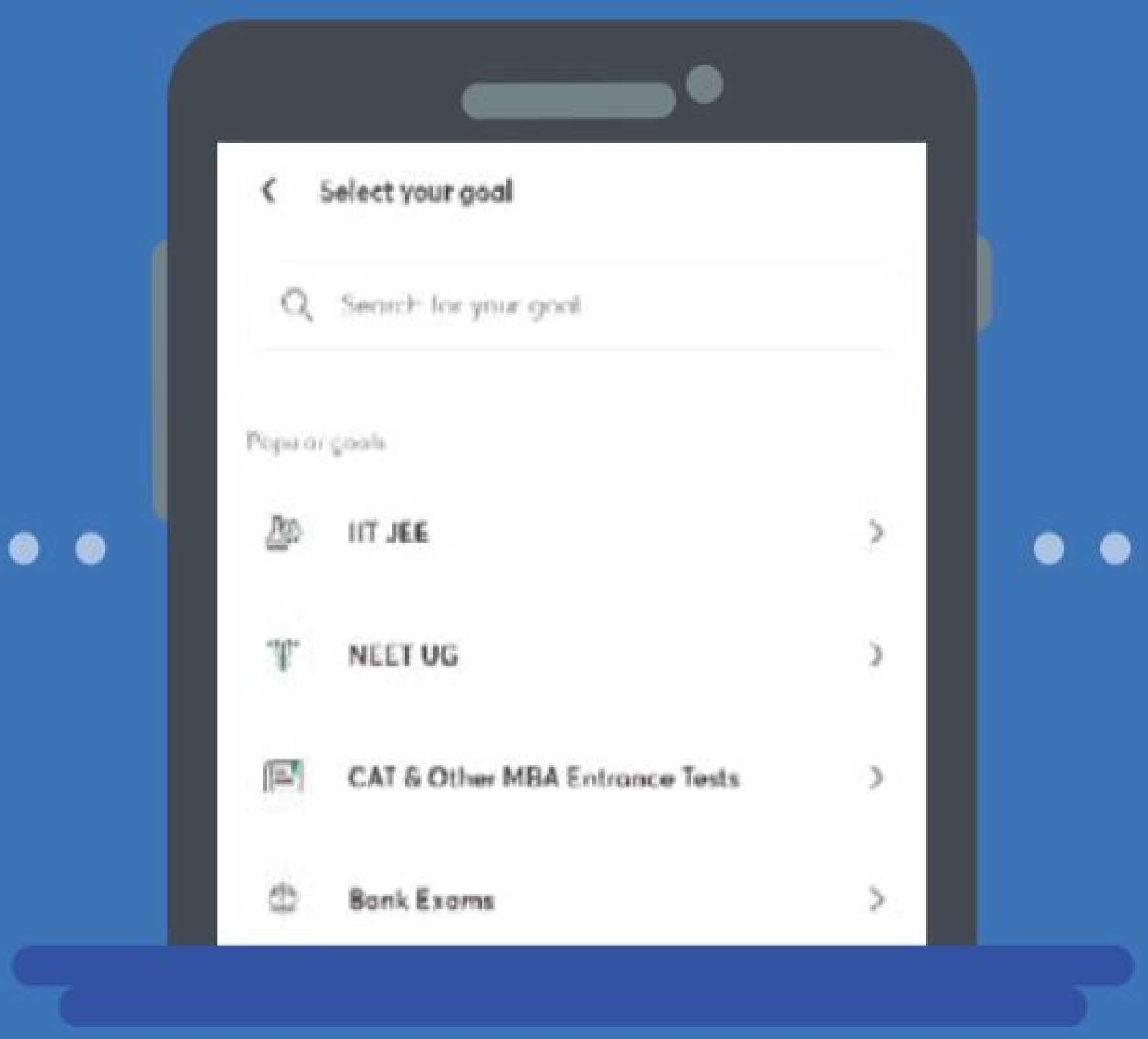
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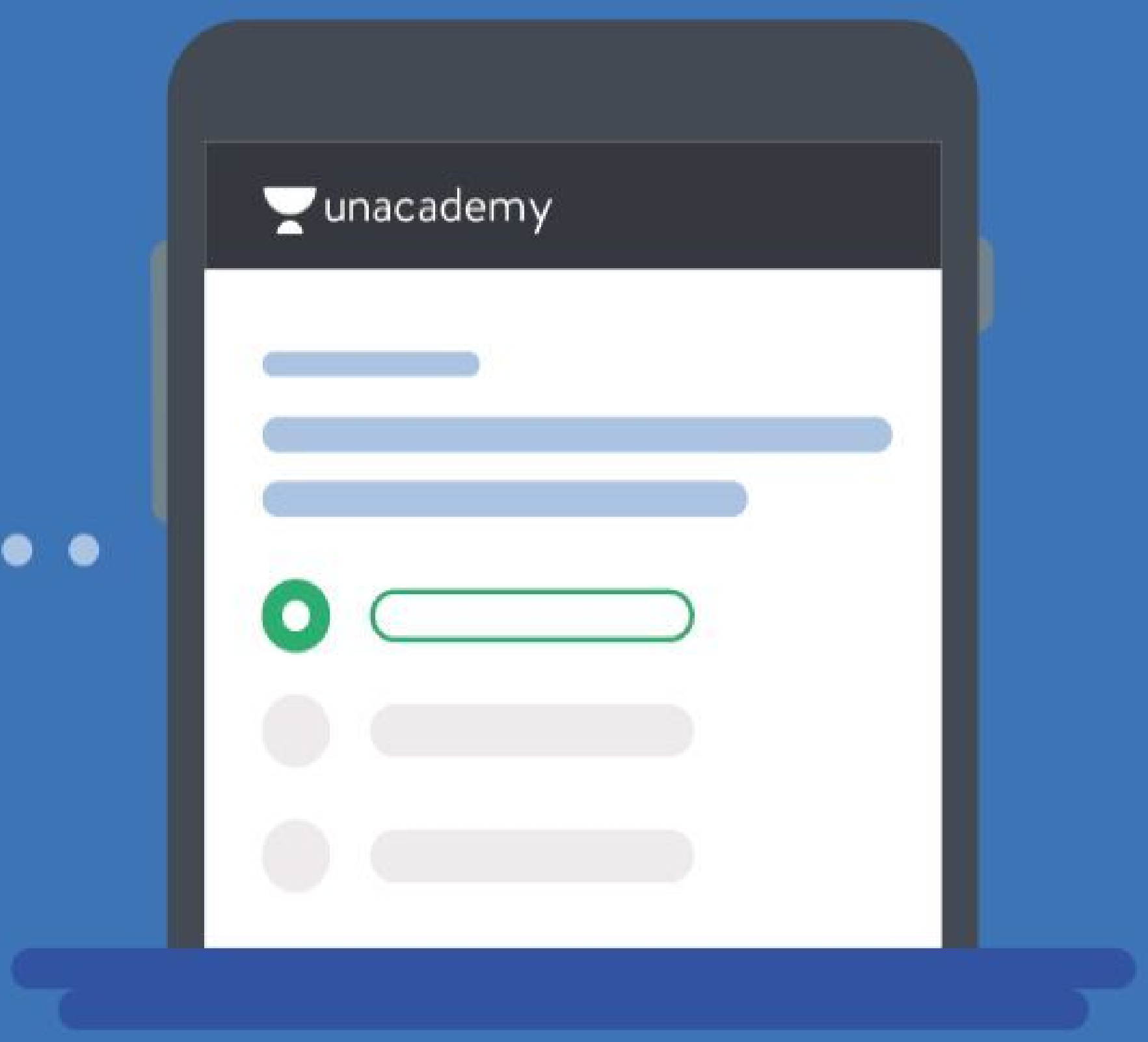

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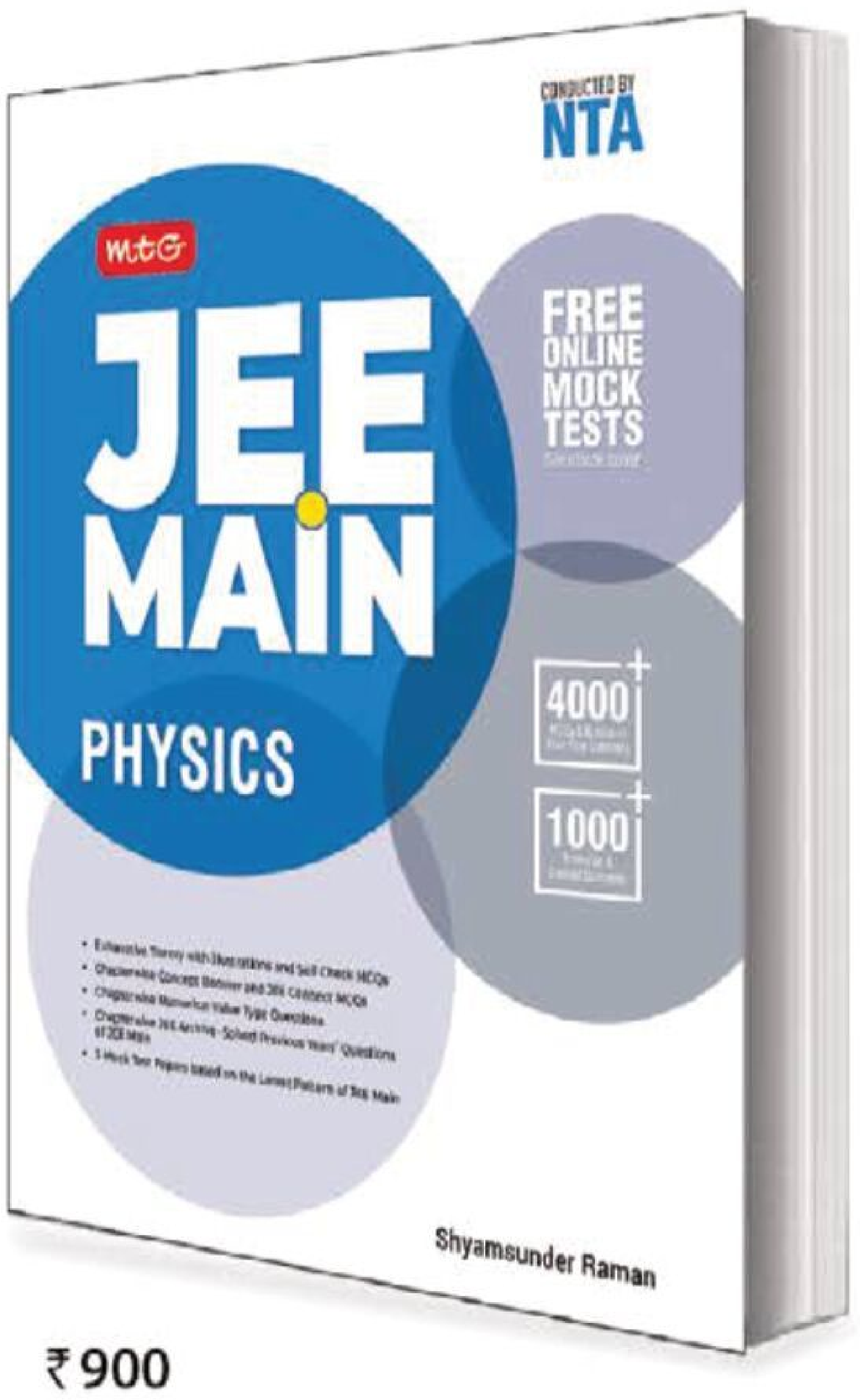
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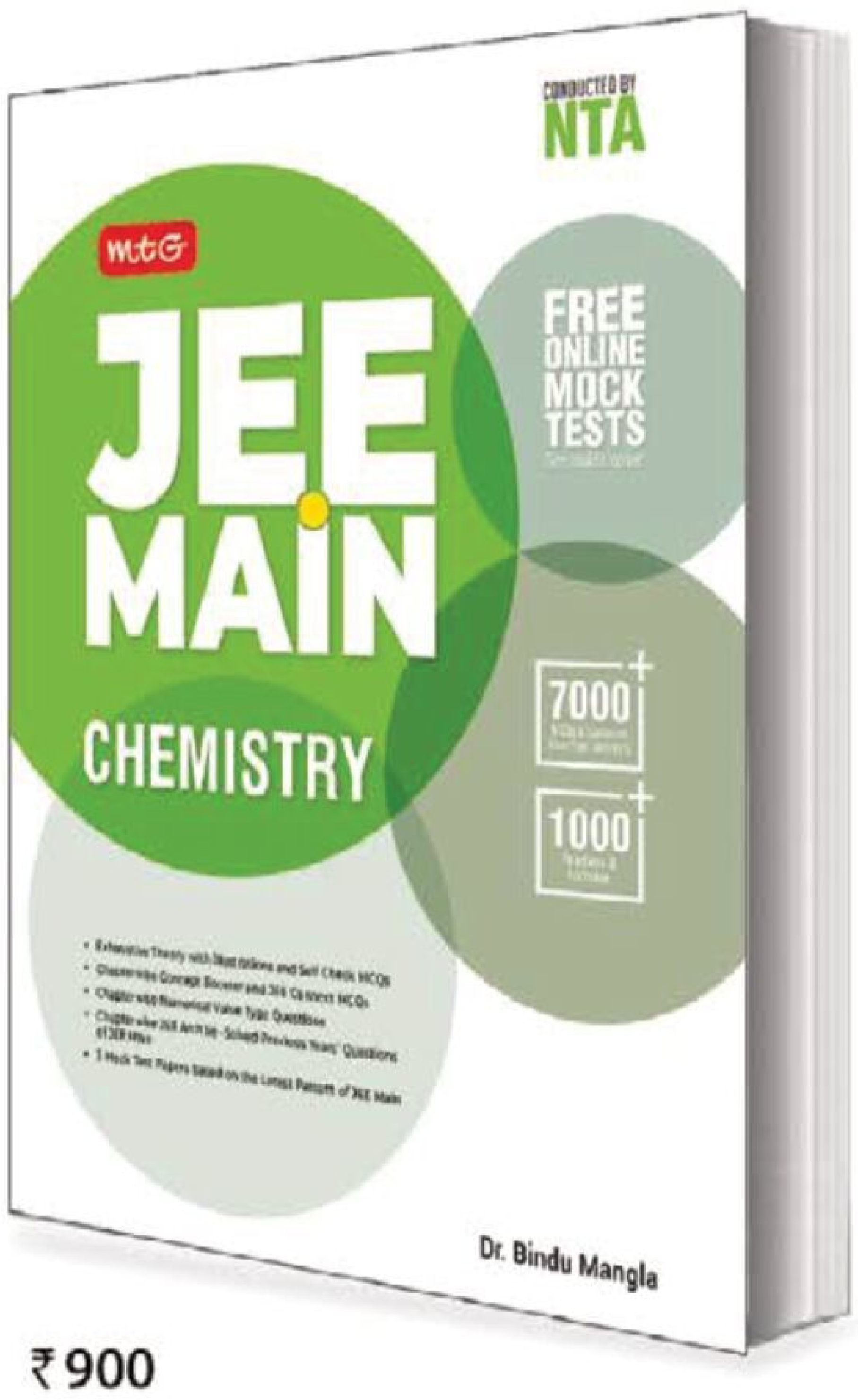
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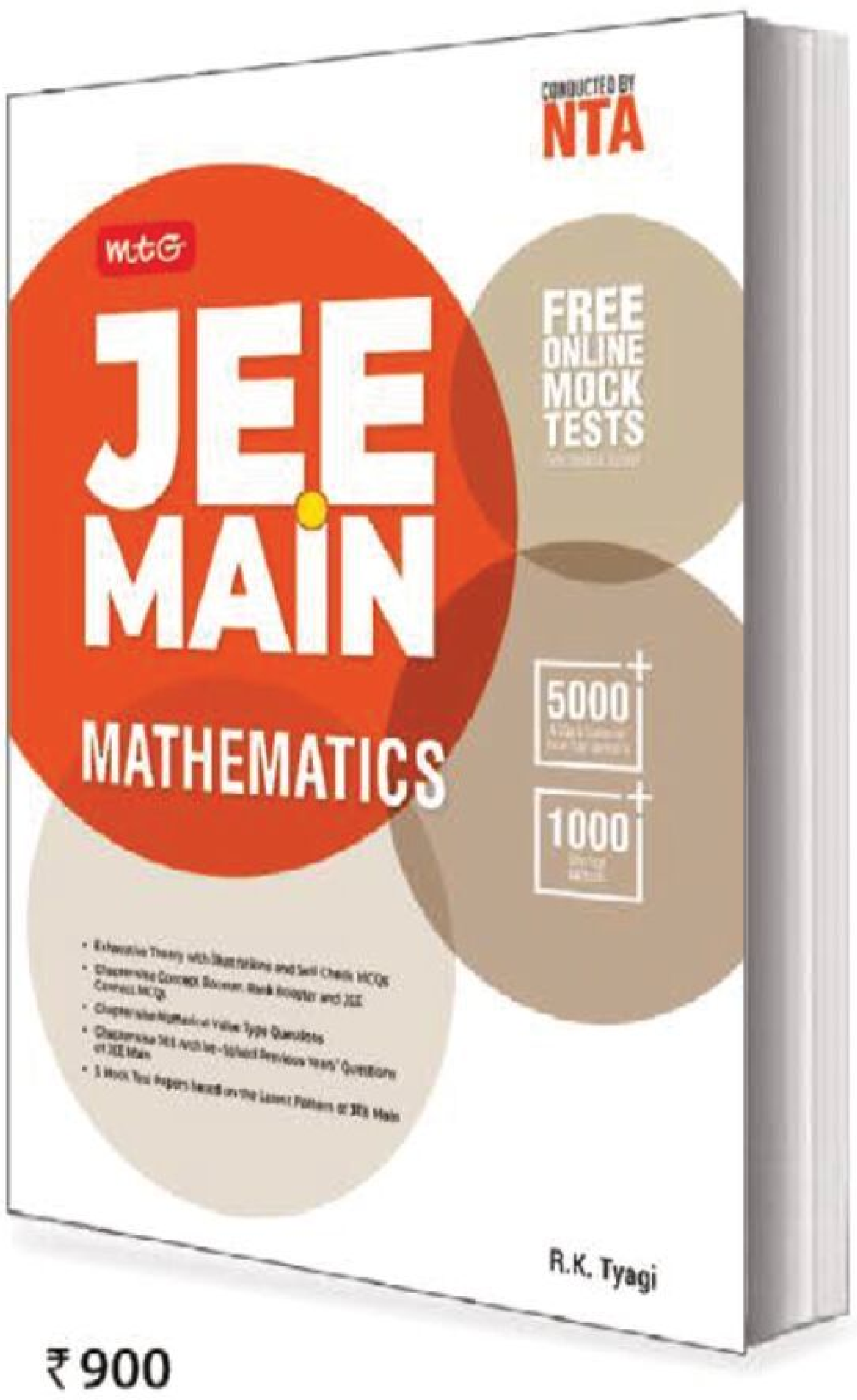
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Vineet Loomba

IITian | Mentor to 100%iler in both Feb & March attempts of JEE Main 2021

JEE Advanced is not only an exam but is also a dream of millions of aspirants to enter the Golden Gates of IITs, the top engineering institute in the country. It is believed to be one of the toughest examinations in the world at the higher secondary level. But with the right strategy and guidance, even a mediocre & hardworking student can crack the exam with ease. Let's discuss the complete strategy & tips which will help you stand out in this exam.

The first step is to choose the right mentors. Research, try them yourself through demo classes and then choose your mentors independently. Once chosen, stick with them till the end. Interact with them regularly, share your problems and find effective solutions.

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Books and study materials

play an equally important role in your preparation. Consult with your chosen mentors as they will be in the best position to help you choose the best books and study materials for you.

Being a civil engineering graduate from IIT Roorkee, Vineet Loomba gave up a promising career in civil engineering to help students prepare for IIT JEE. His experience of more than a decade and his belief in systematic and planned preparation has helped numerous students achieve their dreams. His friendly approach and unconventional teaching style have made him popular amongst the students. Students love him and he loves his students. He is currently the No.1 Educator at Unacademy and through this platform, he is impacting the lives of thousands of Learners on daily basis.

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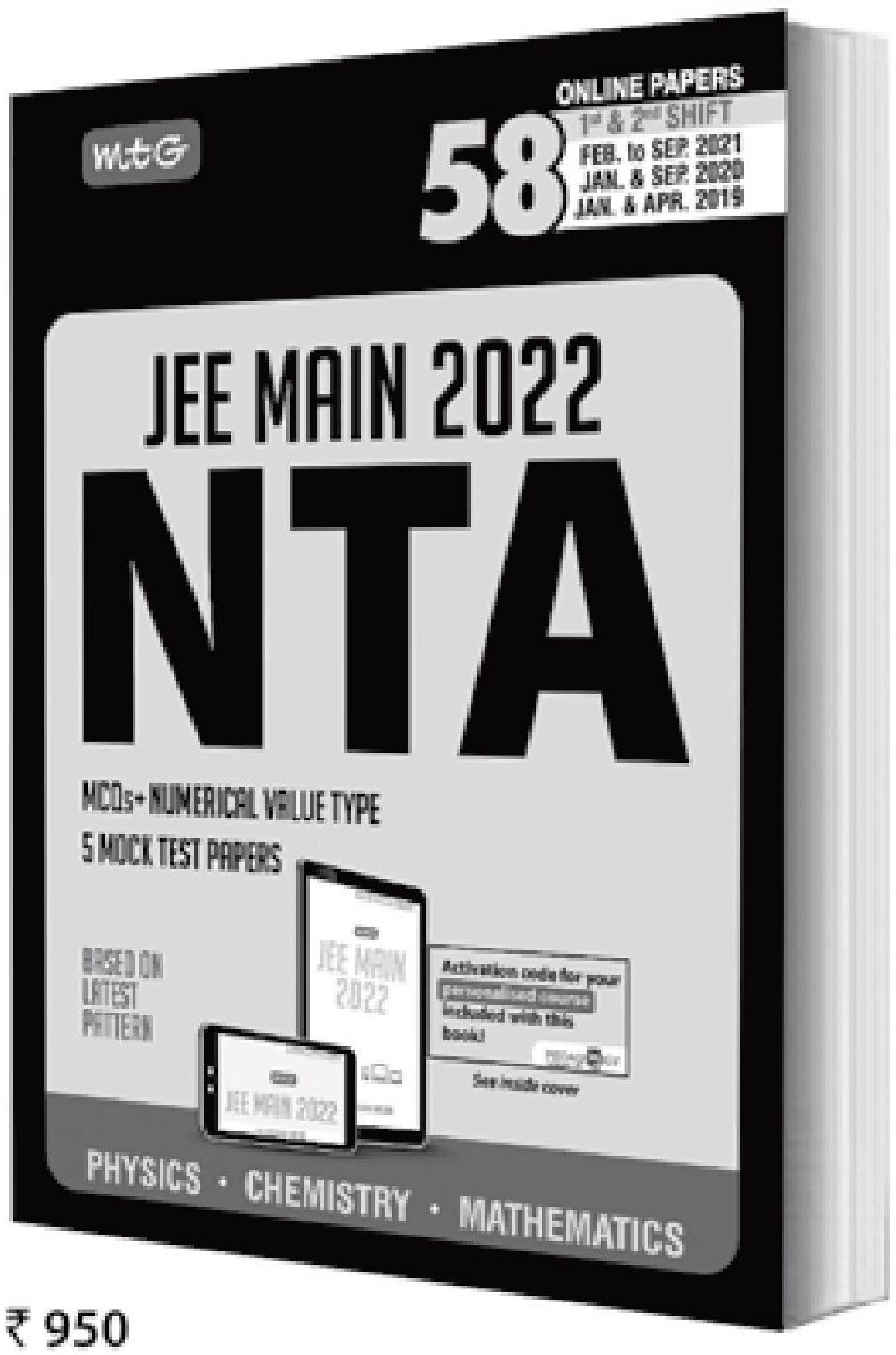
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Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital,
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Editor : Anil Ahlawat

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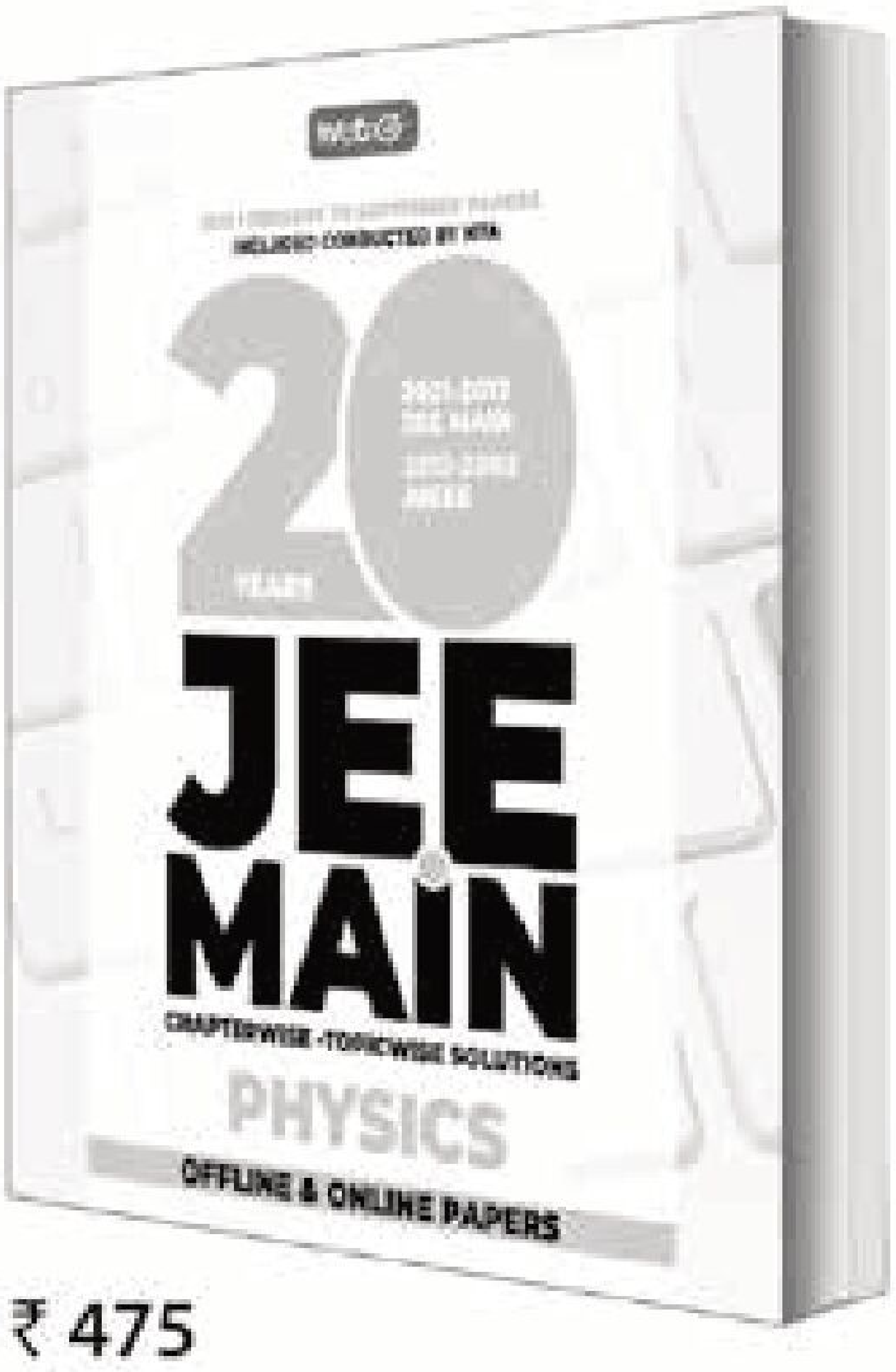

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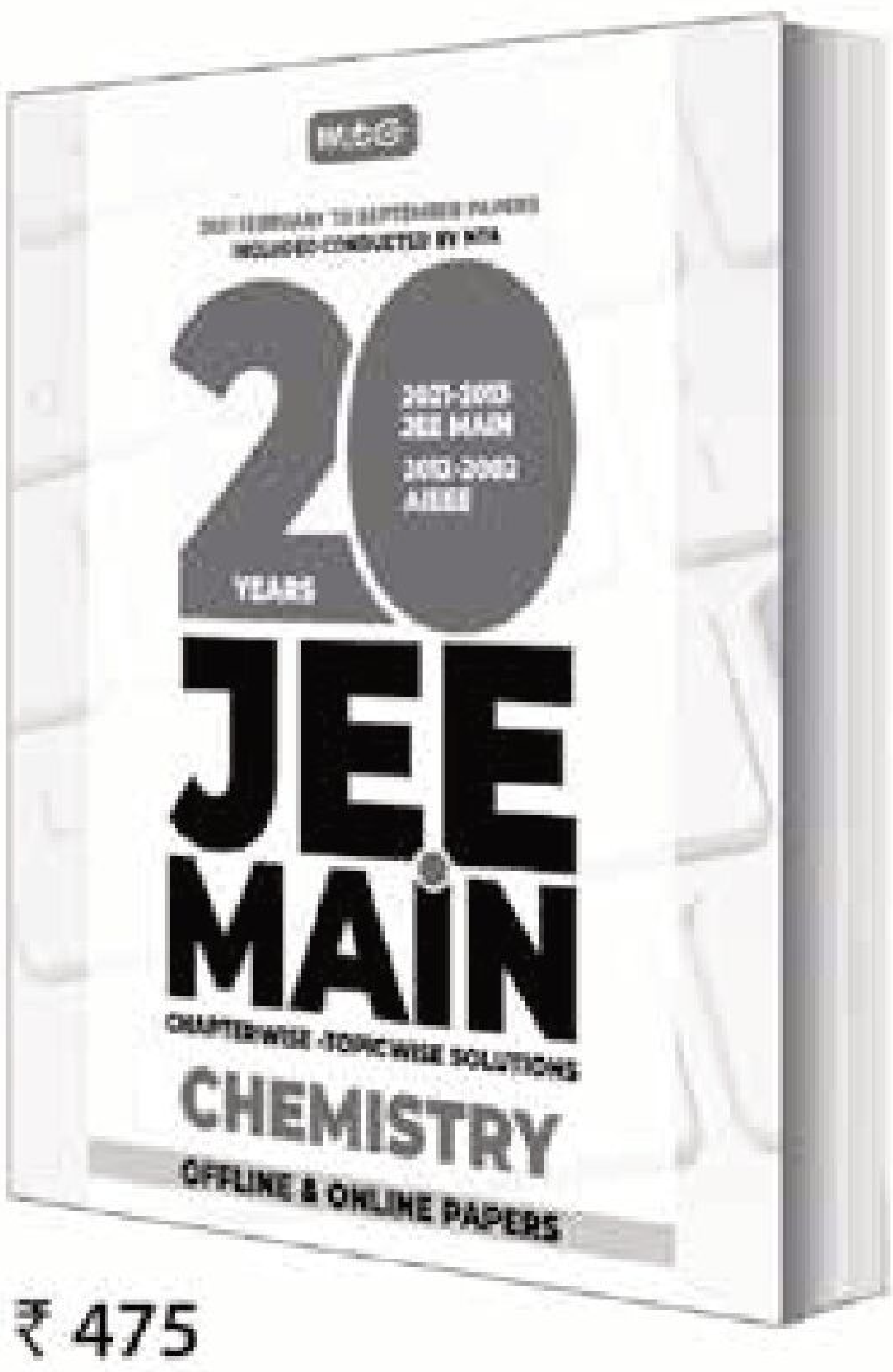

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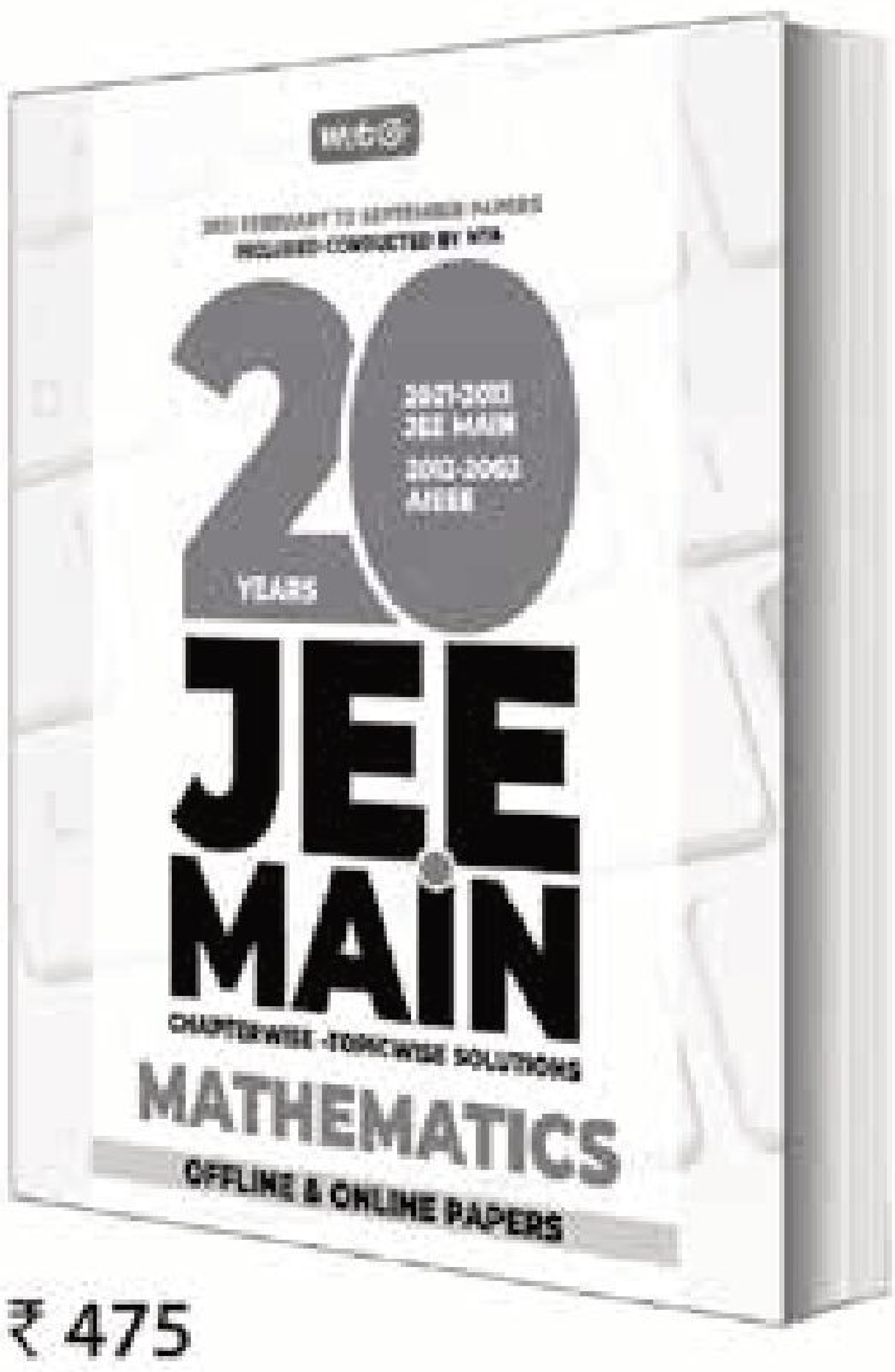

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PRACTICE PAPER 2022
JEE MAIN

Exam Dates
Session-1
23rd to 29th June
Session-2
21st to 30th July

SECTION - A

MULTIPLE CHOICE QUESTIONS

1. Let $f(\theta) = \begin{vmatrix} \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix}$

and the period of $f(\theta)$ is T, then the value of $\frac{4}{\pi}\times T$ is

(a) 2 (b) 3
(c) 4 (d) none of these

2. Let $A = \{x : x \text{ is a prime factor of } 240\}$

$B = \{x : x \text{ is the sum of any two prime factor of } 240\}$. Then $A \cup B$ is

(a) $\{2, 3, 5\}$ (b) $\{2, 3, 5, 7, 8\}$
(c) $\{2, 3, 5, 7\}$ (d) none of these

3. If x_1 and x_2 are the roots of the equation $4^x - 3(2^{x+3}) + 128 = 0$, then the value of $\frac{x_1 + x_2}{|x_1 - x_2|}$ is

(a) 5 (b) 6 (c) 7 (d) 8

4. The number of values of 't' for which there exist a non-zero 3×3 matrix B such that $B' = tB$ is

(a) 0 (b) 1 (c) 2 (d) infinite

5. Let T_n be the number of possible quadrilaterals formed by joining vertices of an n sided regular polygon. If $T_{n+1} - T_n = 20$, then the number of triangles formed by joining the vertices of the polygon is

(a) 15 (b) 16
(c) 20 (d) none of these

6. Coefficient of x^2 in the expansion of

$$E = (x^{2/3} + 4x^{1/3} + 4)^5 \left(\frac{1}{x^{1/3} - 1} + \frac{1}{x^{2/3} + x^{1/3} + 1}\right)^{-9}$$

is

(a) 20 (b) 24
(c) −18 (d) none of these

7. If $t_1, t_2, t_3, \ldots\ldots, t_n, \ldots\ldots$ are in A.P. such that $t_4 - t_7 + t_{10} = p$, then sum of the first 13 terms of the A.P. is

(a) $10p$ (b) $12p$ (c) $13p$ (d) $15p$

8. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots\ldots + 20(21)^{19} = m(20)^{19}$, then the value of $\sqrt{m+41}$ is

(a) 20 (b) 21
(c) 22 (d) none of these

9. If $\lim\limits_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = b$, then value of $2b$ is

(a) 1 (b) 3
(c) 4 (d) none of these

10. If $y = \ln\cos\left(\tan^{-1}\frac{e^x - e^{-x}}{2}\right)$

Statement I : $y'(0) = 0$

Statement II : $y'(x) = \frac{e^x - e^{-x}}{1 + x^2}$

(a) Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
(b) Statement I is True, Statement II is True; Statement II is not a correct explanation for Statement I.
(c) Statement I is True, Statement II is False.
(d) Statement I is False, Statement II is True.

11. The length of the rectangle of the largest area, that can be inscribed in the ellipse $x^2 + 2y^2 = 8$, are given by

(a) $2\sqrt{6}$ (b) $4\sqrt{6}$
(c) $3\sqrt{6}$ (d) none of these

12. If the primitive of $\frac{1}{(e^x - 1)^2}$ is $f(x) - \ln|g(x)| + C$ then $f(x)$ and $g(x)$ are respectively

(a) $(1 - e^x)$ and $1 - e^{-x}$ (b) $(1 - e^x)-1$ and $1 - e^x$
(c) $(1 - e^x)-1$ and $1 - e^{-x}$
(d) none of these

13. If $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} = \frac{\pi}{k^2+11}$, then the value of k is

(a) ± 7 (b) ± 8

(c) ± 9 (d) none of these

14. The solution of $\frac{dy}{dx} = \frac{1}{2x-y^2}$ is given by $x = cg(y) + ay^2 + by + d$, where c, a, b, d are constants, then $a + b + d$ is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{5}{4}$ (d) none of these

15. If a, b, c are in A.P., a, x, b are in G.P. and b, y, c are in G.P., then the locus of the point (x, y) is

(a) a straight line (b) a circle

(c) an ellipse (d) a hyperbola

16. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect at two distinct points P and Q, then the line $5x + by - a = 0$ passes through P and Q for

(a) infinitely many values of a

(b) exactly two values of a

(c) exactly one value of a

(d) no value of a

17. AB is a chord of the parabola $y^2 = 4ax$ with the end A at the vertex of the given parabola. BC is drawn perpendicular to AB meeting the axis of the parabola at C. The projection of BC on the axis of the parabola is

(a) $\frac{1}{4}$ (latus-rectum) (b) semi latus-rectum

(c) latus-rectum (d) twice latus-rectum

18. Consider the two curves $C_1 : y^2 = 4x$,

$C_2 : x^2 + y^2 - 6x + 1 = 0$.

Statement I: C_1 and C_2 touch each other exactly at two points.

Statement II: Equation of the tangent at $(1, 2)$ to C_1 and C_2 both is $x - y + 1 = 0$ and at $(1, -2)$ is $x + y + 1 = 0$.

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

19. Let L_1 be the line $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 be the line $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$. The vector along the normal of the plane containing the lines L_1 and L_2 is

(a) $7\hat{i} - 7\hat{j} + 8\hat{k}$ (b) $7\hat{i} + 7\hat{j} + 8\hat{k}$

(c) $7\hat{i} - 7\hat{j} + 7\hat{k}$ (d) none of these

20. $\sum_{K=1}^{10} \frac{(-1)^{K-1}}{K}.({}^{10}C_K) =$

(a) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{11}$

(b) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{10}$

(c) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{9}$

(d) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{12}$

SECTION - B

NUMERICAL TYPE

Attempt any 5 questions out of 10.

21. The distance between the line $\frac{x-2}{1} = \frac{y+2}{-1} = \frac{z-3}{4}$ and the plane $x + 5y + z = 5$ is $\frac{p}{q\sqrt{q}}$, then $p + q =$____.

22. If $\sum_{i=1}^{18} (x_i - 8) = 9$ and $\sum_{i=1}^{18} (x_i - 8)^2 = 45$, then value of the standard deviation of $x_1, x_2, ..., x_{18}$ is

23. Let $A = \{x, y, z, u\}$ and $B = \{a, b\}$. A function $f : A \to B$ is selected randomly. Probability that function is an onto function is p/q, then the value of $q - p$ is

24. If letters of the word "ASSASSIN" are written down at random in a row, the probability that no two S's occur together is $K/14$, the value of K is _____.

25. If $\cot\alpha$ equals to the integral solution of the inequality $4t^2 - 16t + 15 < 0$ and $\sin\beta$ equals to the slope of the bisector of the second quadrant, the value of $\sin(\alpha + \beta)\sin(\alpha - \beta)$ is equal to λ, then $|\lambda|$ _____.

26. If $\sin^{-1}\left(\frac{a}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of $a^2 + 1$ is _____.

27. The value of $\sin^{-1}(\sin 10) = m\pi + n$, then $m - n =$

28. It is given that complex numbers z_1 and z_2 satisfy $|z_1| = 2$ and $|z_2| = 3$. If the included angle of their corresponding vectors is 60°, then $\left|\dfrac{z_1+z_2}{z_1-z_2}\right|$ can be expressed as $\dfrac{\sqrt{x}}{7}$ where 'x' is a natural number, then 'x' equals to

29. Let p and q are positive integers. f is a function defined for positive numbers and attains only positive values such that $f(xf(y)) = x^p y^q$ and $p^\lambda = q^\mu$, then the value of $\lambda - \mu$ is

30. If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ and $\vec{\delta}$ are unit vectors, then $|\vec{\alpha}-\vec{\beta}|^2 +$ $|\vec{\beta}-\vec{\gamma}|^2 + |\vec{\gamma}-\vec{\delta}|^2 + |\vec{\delta}-\vec{\alpha}|^2 + |\vec{\gamma}-\vec{\alpha}|^2 + |\vec{\beta}-\vec{\delta}|^2$ does not exceed, λ, then the value of λ is

SOLUTIONS

1. (a) : $f(\theta) = \begin{vmatrix} \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix}$

$$= \begin{vmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix}$$

$(R_1 \to R_1 - R_3;\ R_2 \to R_2 - R_3)$

$$= \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1+\sin^2\theta+4\sin 4\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix} \quad (C_1 \to C_1 + C_3)$$

$= -(\cos^2\theta + 1 + \sin^2\theta + 4\sin 4\theta) = -2(1 + 2\sin 4\theta)$

which is periodic function with period $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Hence, the value of $\dfrac{4}{\pi} \times T = \dfrac{4}{\pi}\cdot\dfrac{\pi}{2} = 2$

2. (b): $240 = 2^4 \times 3 \times 5$

So, $A = \{2, 3, 5\}$, $B = \{5, 7, 8\}$

Clearly, $A \cup B = \{2, 3, 5, 7, 8\}$

3. (c) : Put $2^x = y$, given equation becomes

$y^2 - 3(8y) + 128 = 0 \quad \Rightarrow \quad y^2 - 24y + 128 = 0$

$\Rightarrow \quad (y - 8)(y - 16) = 0 \quad \Rightarrow \quad y = 16, 8$

$\Rightarrow \quad 2^x = 16, 8 \Rightarrow x = 4, 3$

Hence $\dfrac{x_1+x_2}{|x_1-x_2|} = \dfrac{4+3}{|4-3|} = 7$

4. (c) : We have, $B = (B')' = (tB)' = tB' = t^2B$

As $B \neq 0$, $t^2 = 1 \Rightarrow t = \pm 1$

5. (c) : T_n = Number of ways of choosing four vertices out of $n = {}^nC_4$

$\therefore \quad 20 = T_{n+1} - T_n = {}^{n+1}C_4 - {}^nC_4$

$= {}^nC_4 + {}^nC_3 - {}^nC_4 = {}^nC_3 \quad [\because\ {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r]$

$\therefore \quad 20 = \dfrac{n(n-1)(n-2)}{3\cdot 2\cdot 1}$

$\Rightarrow \quad n(n-1)(n-2) = 120 = 6.5.4$

$\Rightarrow \quad n = 6$

$\therefore$ Required number of triangles $= {}^nC_3 = {}^6C_3 = 20$.

6. (c) : $E = ((x^{1/3} + 2)^2)^5 \left(\dfrac{x^{2/3} + x^{1/3} + 1 + x^{1/3} - 1}{x-1}\right)^{-9}$

$= (2 + x^{1/3})^{10} \left(\dfrac{x^{2/3} + 2x^{1/3}}{x-1}\right)^{-9}$

$= (2 + x^{1/3})^{10} \dfrac{(x-1)^9}{(x^{1/3})^9 (x^{1/3}+2)^9} = (2 + x^{1/3})x^{-3}(x - 1)^9$

Coefficient of x^2

$= 2$[coefficient of x^5 in the expansion of $x^{-3}(x - 1)^9$]

$= 2(-1)^9$ [coefficient of x^8 in $(1 - x)^9$]

$= 2(-1)\ {}^9C_8\ (-1)8 = -18$

7. (c) : Let d be the common difference of the A.P., then

$p = t_4 - t_7 + t_{10} = t_4 + (t_{10} - t_7)$

$= t_1 + 3d + 3d = t_1 + 6d$

Now, $S_{13} = \dfrac{13}{2}\ [2t_1 + (13 - 1)d] = 13p$

8. (b) : Dividing the given equation by $(20)^{19}$, we get

$m = 1 + 2x + 3x^2 + \ldots\ldots + 20x^{19}$, where $x = \dfrac{21}{20}$.

$$m = \frac{d}{dx}[x + x^2 + \ldots\ldots + x^{20}] = \frac{d}{dx}\left[\frac{x(x^{20}-1)}{x-1}\right]$$

$$= \frac{(x-1)(21x^{20}-1) - x(x^{20}-1)}{(x-1)^2} = \frac{20x^{21} - 21x^{20} + 1}{(x-1)^2}$$

But, $20x^{21} - 21x^{20} = 20\left(\dfrac{21}{20}\right)^{21} - 21\left(\dfrac{21}{20}\right)^{20} = 0$

$$\therefore\ m=\frac{1}{\left(\frac{21}{20}-1\right)^2}=400$$

Hence, $\sqrt{m+41}=\sqrt{441}=21$

9. (b): $b=\lim\limits_{x\to 0}\dfrac{e^{x^2}-1+1-\cos x}{x^2}$

$$=\lim_{x\to 0}\left(\frac{e^{x^2}-1}{x^2}+\frac{2\sin^2 x/2}{4(x/2)^2}\right)=1+\frac{1}{2}=\frac{3}{2}$$

Hence, $2b = 2(3/2) = 3$

10. (c): $y'(x)=-\dfrac{\sin\left(\tan^{-1}\dfrac{e^x-e^{-x}}{2}\right)}{\cos\left(\tan^{-1}\dfrac{e^x-e^{-x}}{2}\right)}$

$$\times\frac{1}{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}\times\frac{e^x+e^{-x}}{2}$$

$$=-\tan\left(\tan^{-1}\frac{e^x-e^{-x}}{2}\right)\times\frac{2}{(e^x+e^{-x})^2}\times(e^x+e^{-x})$$

$$=-\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$\therefore\ y'(0) = 0$

11. (a): Any point on the ellipse $\dfrac{x^2}{8}+\dfrac{y^2}{4}=1$ is $(2\sqrt{2}\cos\theta, 2\sin\theta)$.

A = Area of the inscribed rectangle

$= 4(2\sqrt{2}\cos\theta)(2\sin\theta)=8\sqrt{2}\sin 2\theta$

$$\frac{dA}{d\theta}=16\sqrt{2}\cos 2\theta=0 \Rightarrow \theta=\pi/4$$

Also, $\dfrac{d^2A}{d\theta^2}=-32\sqrt{2}\sin 2\theta<0$ for $\theta=\dfrac{\pi}{4}$.

Hence, the inscribed rectangle is of the largest area if the sides are

$4\sqrt{2}\cos\dfrac{\pi}{4}$ and $4\sin\left(\dfrac{\pi}{4}\right)$ *i.e.*, 4 and $2\sqrt{2}$.

Hence, the length of diagonal

$$=\sqrt{4^2+(2\sqrt{2})^2}=\sqrt{16+8}=\sqrt{24}=2\sqrt{6}$$

12. (c): $\displaystyle\int\frac{1}{(e^x-1)^2}dx=\int\frac{e^x dx}{e^x(e^x-1)^2}=\int\frac{dt}{t(t-1)^2}$

$$=\int\frac{1}{t-1}\left[\frac{1}{t-1}-\frac{1}{t}\right]dt=\int\left(\frac{1}{(t-1)^2}-\frac{1}{t-1}+\frac{1}{t}\right)dt$$

$$\left[\because\ \text{Let } t=e^x \Rightarrow dt=e^x dx\right]$$

$=\dfrac{1}{1-t}-\ln|t-1|+\ln|t|+C$

$=(1-e^x)^{-1}-\ln|1-e^{-x}|+C=f(x)-\ln|g(x)|+C$

Hence, $f(x) = (1 - e^x)^{-1}$ and $g(x) = 1 - e^{-x}$

13. (a): $\displaystyle\int_0^\infty\frac{dx}{(x^2+4)(x^2+9)}=\frac{1}{5}\int_0^\infty\left[\frac{1}{x^2+4}-\frac{1}{x^2+9}\right]dx$

$$=\frac{1}{5}\left[\frac{1}{2}\tan^{-1}\frac{x}{2}-\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^\infty=\frac{1}{5}\left[\frac{\pi}{4}-\frac{\pi}{6}\right]=\frac{\pi}{60}$$

So, $k^2 + 11 = 60$

$\Rightarrow\ k^2 = 49 \Rightarrow k = \pm 7$

14. (c): We have, $\dfrac{dx}{dy}=2x-y^2$

This is a linear equation in x.

The integrating factor is $e^{-\int 2dy}=e^{-2y}$

So, $\dfrac{d}{dy}(xe^{-2y})=-y^2e^{-2y}$

Integrating, we have

$$xe^{-2y}=\frac{-y^2e^{-2y}}{-2}-\int ye^{-2y}dy+C$$

$$=\frac{y^2}{2}e^{-2y}+\frac{ye^{-2y}}{2}-\frac{1}{2}\int e^{-2y}dy+C$$

$$=\frac{y^2}{2}e^{-2y}+\frac{y}{2}e^{-2y}+\frac{e^{-2y}}{4}+C$$

$$\therefore\ x=ce^{2y}+\frac{y^2}{2}+\frac{y}{2}+\frac{1}{4}$$

On comparing with the above equation, we get

$$a=\frac{1}{2}, b=\frac{1}{2}, d=\frac{1}{4}$$

Hence, $a+b+d=\dfrac{5}{4}$.

15. (b): We have, $2b = a + c$, $x^2 = ab$ and $y^2 = bc$

$\therefore\ x^2 + y^2 = b(a + c) = 2b^2$, which is a circle.

16. (d) : Equation of the line passing through the points P and Q is

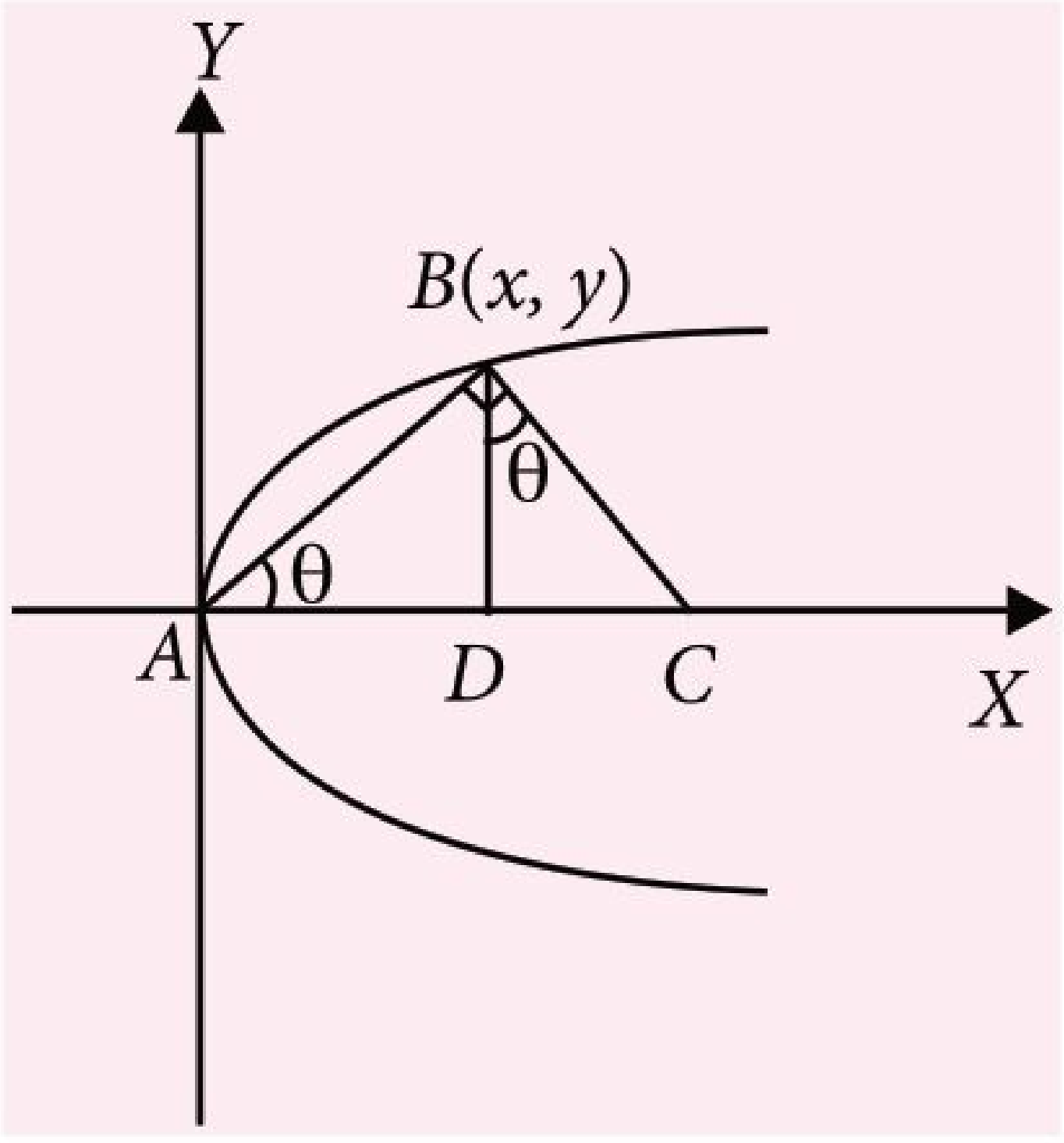
$x^2 + y^2 + 2ax + cy + a - (x^2 + y^2 - 3ax + dy - 1) = 0$

$\Rightarrow\ 5ax + (c - d)y + a + 1 = 0$

which coincides with $5x + by - a = 0$ if $\dfrac{5a}{5} = \dfrac{c-d}{b} = \dfrac{a+1}{-a}$

or if $a^2 + a + 1 = 0$ which does not give any real value of a.

17. (c) : Draw BD perpendicular to the axis of the parabola.

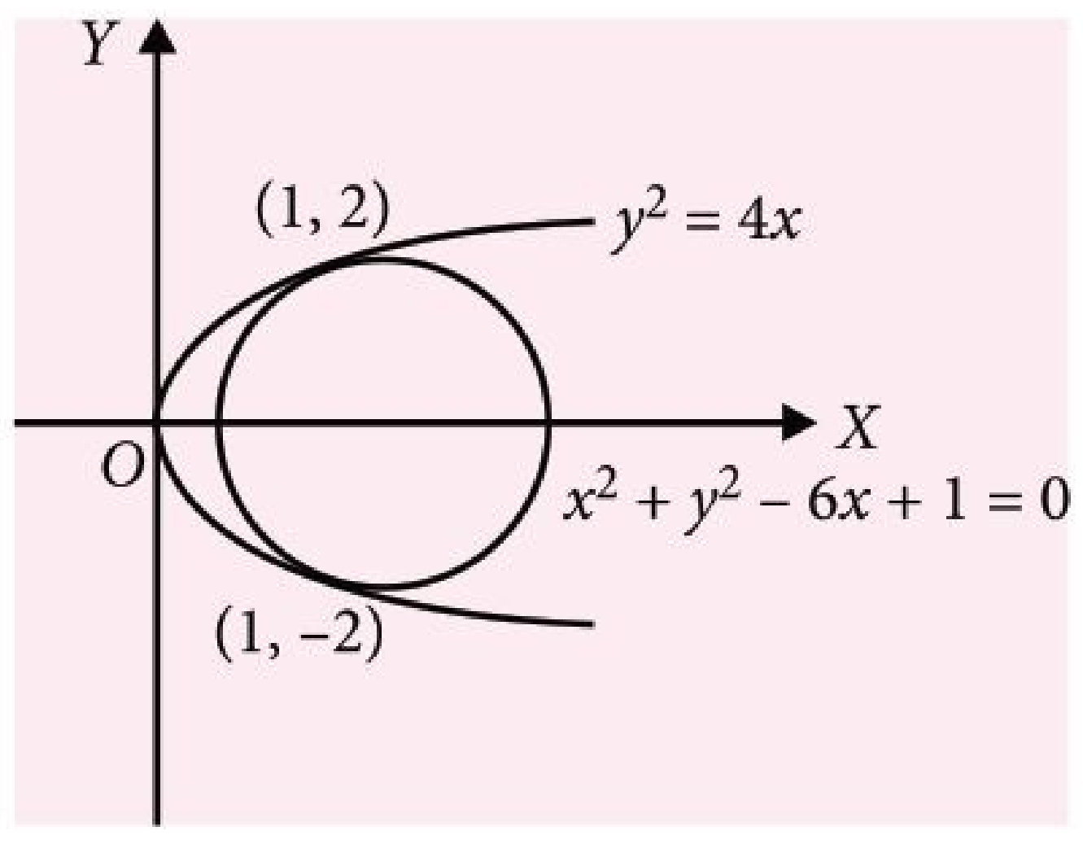


Let the coordinates of B be (x, y) then slope of AB is given by $\tan\theta = \dfrac{y}{x}$

Projection of BC on the axis of the parabola is $DC = BD\tan\theta$

$$= y\left(\frac{y}{x}\right) = \frac{y^2}{x} = \frac{4ax}{x} = 4a$$

18. (a) : Solving for the points of intersection, we have



$x^2 + 4x - 6x + 1 = 0$

$\Rightarrow\ (x - 1)^2 = 0$

$\Rightarrow\ x = 1$

$\Rightarrow\ y = \pm 2$

Thus, the two curves meet at (1, 2) and (1, –2).

Tangent at (1, 2) to $C_1 : y^2 = 4x$ is $y(2) = 2(x + 1)$

$\Rightarrow\ x - y + 1 = 0$

Tangent at (1, 2) to $C_2 : x^2 + y^2 - 6x + 1 = 0$ is

$x + 2y - 3(x + 1) + 1 = 0$

or $x - y + 1 = 0$ same as the tangent to the curve C_1.

Similarly, the tangent at the point (1, –2) to the two curves is $x + y + 1 = 0$

$\Rightarrow$ Statement II is true and hence, Statement I is also true.

19. (a) : The plane through L_1 is

$2x - 2y + 3z - 2 + \lambda(x - y + z + 1) = 0$

and the plane through L_2 is

$x + 2y - z - 3 + \mu(3x - y + 2z - 1) = 0$

Two planes are same.

So, $\dfrac{2+\lambda}{1+3\mu} = \dfrac{2+\lambda}{\mu-2} = \dfrac{3+\lambda}{2\mu-1} = \dfrac{2-\lambda}{3+\mu}$

$\Rightarrow\ \mu = -\dfrac{3}{2}$ and $\lambda = 5$ and the equation of the plane is

$7x - 7y + 8z + 3 = 0.$

Hence, the vector along the normal is $7\hat{i} - 7\hat{j} + 8\hat{k}$.

20. (b) : Required value is

$$\frac{{}^{10}C_1}{1} - \frac{{}^{10}C_2}{2} + \frac{{}^{10}C_3}{3} \ldots\ldots - \frac{{}^{10}C_{10}}{10}$$

To find which, consider

$$(1 - x)^{10} = {}^{10}C_0 - {}^{10}C_1x + {}^{10}C_2x^2 \ldots. + {}^{10}C_{10}x^{10}$$

$$\Rightarrow\ \frac{(1-x)^{10}-1}{x} = -[{}^{10}C_1 - {}^{10}C_2x + \ldots\ldots. - {}^{10}C_{10}\,x^9]$$

$$\Rightarrow\ \int_0^1 \frac{1-(1-x)^{10}}{x}\,dx = \int_0^1 [{}^{10}C_1 - {}^{10}C_2x + \ldots. - {}^{10}C_{10}\,x^9]\,dx$$

$$= {}^{10}C_1 - \frac{{}^{10}C_2}{2} + \frac{{}^{10}C_3}{3} - \ldots. - \frac{{}^{10}C_{10}}{10}$$

To find L.H.S. consider

$$I_n = \int_0^1 \frac{1-(1-x)^n}{x}\,dx \ \Rightarrow\ I_{n+1} - I_n = \int_0^1 (1-x)^n dx = \frac{1}{n+1}$$

$$\therefore\ I_{n+1} = \frac{1}{n+1} + I_n$$

$$\therefore\ I_{10} = \int_0^1 \frac{1-(1-x)^{10}}{x}dx = \frac{1}{10} + I_9 = \frac{1}{10} + \frac{1}{9} + I_8 \approx \ldots..$$

$$= \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + \ldots\ldots + 1$$

21. (13) : Direction ratios of the line are 1, –1, 4 and that of the normal to the plane are 1, 5, 1.

Since, $1 \times 1 + (-1)(5) + 4 \times 1 = 0$

$\therefore$ Normal to the plane is perpendicular to the line and thus, the line is parallel to the plane and the distance between them is the distance of any point, let (2, –2, 3) on the line from the plane and is equal to

$$\left|\frac{2(1)+(-2)(5)+3\times1-5}{\sqrt{1+25+1}}\right| = \frac{10}{\sqrt{27}} = \frac{10}{3\sqrt{3}}$$

$$\therefore \frac{10}{3\sqrt{3}} = \frac{p}{q\sqrt{q}} \quad \text{(Given)}$$

$\Rightarrow p + q = 10 + 3 = 13$

22. (1.5) : Let $d_i = x_i - 8$ but

$$\sigma_x^2 = \sigma_d^2 = \frac{1}{18}\sum d_i^2 - \left(\frac{1}{18}\sum d_i\right)^2$$

$$= \frac{1}{18} \times 45 - \left(\frac{9}{18}\right)^2 = \frac{5}{2} - \frac{1}{4} = \frac{9}{4}$$

Therefore $\sigma_x = 3/2$

23. (1) : Total number of functions from A to B is $2^4 = 16$. There are exactly two functions which are not onto viz. one in which all the elements of A are mapped to a and other in which all the elements of A are mapped to b. Thus, there are 14 onto functions. Therefore probability of required event is $\frac{14}{16}=\frac{7}{8}=\frac{p}{q}$ (Given)

Hence, $q - p = 1$

24. (1) : The number of ways of permuting the letters of the word "ASSASSIN" is

$$\frac{8!}{2!4!}=\frac{8\times7\times6\times5}{2}=840$$

To find the number of ways n which two S's are not together,

We first arrange A, A, I, N. This can be done in

$\frac{4!}{2!}$ = 12 ways.

In the following figure 4S's can be arranged at four boxes out of the five boxes.

□ × □ × □ × □ × □

Thus, corresponding to each arrangement of A, A, I, N, there are ${}^5C_4 = 5$ ways of arranging S's.

$\therefore$ The number of favourable ways = (12) × (5) = 60

Thus, required probability $=\frac{60}{840}=\frac{1}{14}=\frac{K}{14}$ (Given)

Hence, $K = 1$

25. (0.8) : We have, $4t^2 - 16t + 15 < 0 \Rightarrow \frac{3}{2}<t<\frac{5}{2}$

$\Rightarrow$ $\cot\alpha = 2$, the integral solution of the given inequality and $\sin\beta = \tan 135° = -1$

$\therefore$ $\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta$

$$=\frac{1}{1+\cot^2\alpha}-1=\frac{1}{1+4}-1=-\frac{4}{5}=\lambda \qquad \text{(Given)}$$

Hence, $|\lambda|=\frac{4}{5}=0.8$

26. (10) : $\sin^{-1}\left(\frac{a}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left(\frac{a}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\frac{a}{5}\right)+\cos^{-1}\sqrt{1-\frac{16}{25}}=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\frac{a}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\frac{\pi}{2}$$

$\Rightarrow$ $a = 3$ as $\sin^{-1} a + \cos^{-1} a = \frac{\pi}{2}$ $\forall a$

Hence, $a^2 + 1 = 10$

27. (13) : Let $y = \sin^{-1}(\sin 10)$

$\Rightarrow$ $\sin y = \sin 10 = \sin(3\pi + (10 - 3\pi))$

$= -\sin(10 - 3\pi) = \sin(3\pi - 10)$

$\Rightarrow$ $y = 3\pi - 10 = m\pi + n$ (Given)

$\therefore$ $m = 3, n = -10 \Rightarrow m - n = 13.$

28. (133) : By cosine rule,

$$|z_1+z_2| = \sqrt{|z_1|^2+|z_2|^2+2|z_1||z_2|\cos 60°}=\sqrt{19}$$

$$|z_1-z_2| = \sqrt{|z_1|^2+|z_2|^2-2|z_1||z_2|\cos 60°}=\sqrt{7}$$

$$\left|\frac{z_1+z_2}{z_1-z_2}\right|=\sqrt{\frac{19}{7}}=\frac{\sqrt{133}}{7}=\frac{\sqrt{x}}{7} \qquad \text{(Given)}$$

Hence, x equals to 133.

29. (1) : For $x=\frac{1}{f(y)}$ $\Rightarrow$ $f(1)=\frac{y^q}{(f(y))^p}$

$$\Rightarrow f(y)=\frac{y^{q/p}}{(f(1))^{1/p}},\ y = 1$$

$\Rightarrow$ $f(1) = 1 \Rightarrow f(y) = y^{q/p}$

$f(xy^{q/p}) = x^p y^q$

For $y = z^{p/q}$ $\Rightarrow$ $f(xz) = x^p z^p$

$\Rightarrow$ $f(x) = x^p$

$\frac{q}{p}=p \Rightarrow q=p^2$ and $p^\lambda = q^\mu$ (Given)

On comparing, we get

$\lambda = 2$ and $\mu = 1$

Hence, $\lambda - \mu = 1.$

30. (16) : We have

$$0\le|\vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}|^2=|\vec{\alpha}|^2+|\vec{\beta}|^2+|\vec{\gamma}|^2+|\vec{\delta}|^2 +2(\vec{\alpha}\cdot\vec{\beta}+\vec{\beta}\cdot\vec{\gamma}+\vec{\gamma}\cdot\vec{\alpha}+\vec{\alpha}\cdot\vec{\delta}+\vec{\gamma}\cdot\vec{\delta}+\vec{\beta}\cdot\vec{\delta})$$

$$=4+2(\vec{\alpha}\cdot\vec{\beta}+\vec{\beta}\cdot\vec{\gamma}+\vec{\gamma}\cdot\vec{\alpha}+\vec{\alpha}\cdot\vec{\delta}+\vec{\gamma}\cdot\vec{\delta}+\vec{\beta}\cdot\vec{\delta})$$

So, $\vec{\alpha}\cdot\vec{\beta}+\vec{\beta}\cdot\vec{\gamma}+\vec{\gamma}\cdot\vec{\alpha}+\vec{\alpha}\cdot\vec{\delta}+\vec{\gamma}\cdot\vec{\delta}+\vec{\beta}\cdot\vec{\delta}\ge-2$

Now, $|\vec{\alpha}-\vec{\beta}|^2+|\vec{\beta}-\vec{\gamma}|^2+|\vec{\gamma}-\vec{\delta}|^2+|\vec{\delta}-\vec{\alpha}|^2 +|\vec{\gamma}-\vec{\alpha}|^2+|\vec{\beta}-\vec{\delta}|^2$

$$=3(|\vec{\alpha}|^2+|\vec{\beta}|^2+|\vec{\gamma}|^2+|\vec{\delta}|^2) -2(\vec{\alpha}\cdot\vec{\beta}+\vec{\gamma}\cdot\vec{\delta}+\vec{\delta}\cdot\vec{\alpha}+\vec{\alpha}\cdot\vec{\gamma}+\vec{\beta}\cdot\vec{\delta}+\vec{\beta}\cdot\vec{\gamma})\le 3(4)-2(-2)=16$$

$\therefore$ $\lambda=16$

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Exam Dates
Session-1
23rd to 29th June
Session-2
21st to 30th July

JEE Main 2022

MOCK TEST PAPER

SECTION - A

MULTIPLE CHOICE QUESTIONS

1. An ellipse has eccentricity 1/2 and one focus at $S\left(\frac{1}{2}, 1\right)$. Its one directrix is the common tangent (nearer to S) to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. The equation of the ellipse in standard form is

   (a) $9\left(x-\frac{1}{3}\right)^2+12(y-1)^2=1$

   (b) $12\left(x-\frac{1}{3}\right)^2+9(y-1)^2=1$

   (c) $\dfrac{\left(x-\frac{1}{2}\right)^2}{12}+\dfrac{(y-1)^2}{9}=1$

   (d) $3\left(x+\frac{1}{2}\right)^2+4(y-1)^2=1$

2. AB is a chord of the parabola $y^2 = 4ax$ with vertex at A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the x-axis is

   (a) a (b) $2a$
   (c) $4a$ (d) $8a$

3. If both roots of $x^2 - 2ax + a^2 - 1 = 0$ lies in $(-2, 1)$ then $[a]$, where $[\cdot]$ denotes greatest integral function is

   (a) -1 (b) 0 (c) 1 (d) 2

4. Let R be a relation in N defined by $R = \{(x, y) : 2x + y = 8\}$, then range of R is

   (a) $\{1, 2, 3\}$ (b) $\{2, 4, 6\}$
   (c) $\{1, 2, 3, 4, 6\}$ (d) none of these

5. Let $f(x)=\frac{\sin 4\pi[x]}{1+[x]^2}$, where $[x]$ is the greatest integer, then

   (a) $f(x)$ is not differentiable
   (b) $f'(x) > 0$
   (c) $f'(x) = 0\ \forall\ x$
   (d) none of these

6. If $z \neq 0$, $\int_0^{100} \arg(-|z|)dx$ equals

   (a) 0 (b) not defined
   (c) 100 (d) 100π

7. Sum of the series

   $$\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+\ldots.+\frac{1}{(n-1)!1!}$$ is

   (a) $\frac{2^{n-1}}{(n-1)!}$ (b) $\frac{2^{n-1}}{n!}$ (c) $\frac{2^n}{(n-1)!}$ (d) $\frac{2^n}{n!}$

8. In a 'multiple choice question' test there are eight questions. Each question has four alternatives of which only one is correct. If a candidate answers all the questions by choosing one answer for each question, then the number of ways to get exactly 4 correct answer is

   (a) 70 (b) 2835
   (c) 5670 (d) none of these

9. There are two girls Rina and Tina having m and n distinct number of toys respectively. The number of ways in which they can exchange their toys so that after exchange they have the same number of toys with them but not the same toys, is

   (a) ${}^{m+n}C_m$ (b) ${}^{m+n}C_m - 1$
   (c) ${}^{m+n}P_n$ (d) none of these

10. If the roots of $x^2 + 2ax + b = 0$ are real and they differ by almost $2m$, then b lies in the interval

(a) $(a^2 - m^2, a^2)$ (b) $(a^2, a^2 + m^2)$

(c) $[a^2 - m^2, a^2]$ (d) $(a^2 - m^2, a^2 + m^2)$

11. If $\frac{1+3p}{4}, \frac{1-p}{3}, \frac{1-3p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is

(a) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ (b) $\left[-\frac{1}{3}, 1\right]$

(c) $\left[\frac{1}{13}, 1\right]$ (d) $\left[\frac{1}{13}, \frac{1}{3}\right]$

12. $f(x) = \begin{cases} 3[x] - \frac{5|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$. Then $\int_{-3/2}^{2} f(x)dx =$

([·] denotes the greatest integer function)

(a) $-\frac{11}{2}$ (b) $-\frac{7}{2}$ (c) -6 (d) $-\frac{17}{2}$

13. In a triangle $\angle A > \angle B$ and $\angle A$ and $\angle B$ satisfy the equation $3\sin\theta - 4\sin^3\theta - k = 0, 0 < k < 1$. Then $\angle C =$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

14. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$ such that f is invertible, then its inverse is

(a) $g(y) = 4 + \frac{y+3}{4}$ (b) $g(y) = \frac{y+3}{4}$

(c) $g(y) = \frac{3y+4}{3}$ (d) $g(y) = \frac{y-3}{4}$

15. Area bounded by $|x - 1| \leq 2$ and $x^2 - y^2 = 1$ is

(a) $6\sqrt{3} + \frac{1}{2}\log|3+2\sqrt{2}|$

(b) $6\sqrt{2} + \frac{1}{2}\log|3-2\sqrt{2}|$

(c) $6\sqrt{2} - \log|3+2\sqrt{2}|$

(d) none of these

16. Numbers 1, 2, 3, ..., $2n$ ($n \in N$) are printed on $2n$ cards. The probability of drawing a number r is proportional to r. Then the probability of drawing an even number in one draw is

(a) $\frac{n+2}{n+3}$ (b) $\frac{n+1}{n+3}$

(c) $\frac{1}{2}$ (d) $\frac{n+1}{2n+1}$

17. It is given that event A and B are such that $P(A) = \frac{1}{4}$, $P(B|A) = \frac{2}{3}$, $P(A|B) = \frac{1}{2}$, then $P(B) =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$

18. From the matrix equation $AB = AC$, we say $B = C$ provided

(a) A is singular (b) A is skew symmetric

(c) A is non-singular (d) none of these

19. If $f(x) = \begin{vmatrix} \sin x & 1 & 0 \\ 1 & 2\sin x & 1 \\ 0 & 1 & 2\sin x \end{vmatrix}$ then $\int_{-\pi/2}^{\pi/2} f(x)dx =$

(a) 0 (b) -1 (c) 1 (d) $\frac{3\pi}{2}$

20. The given expression

$$f(x) = \frac{1}{\tan x + \cot x + \sec x + \operatorname{cosec} x}$$

is equivalent to

(a) $\frac{1}{2(\sin x + \cos x - 1)}$ (b) $\frac{\sin x + \cos x - 1}{2}$

(c) $\frac{1}{2(\sin x - \cos x + 1)}$ (d) $\frac{\sin x - \cos x + 1}{2}$

SECTION - B

NUMERICAL VALUE TYPE

Attempt any 5 questions out of 10.

21. Let $f(x) = e^{\cos^{-1}\sin\left(x + \frac{\pi}{3}\right)}$, then $f\left(\frac{8\pi}{9}\right) =$

22. If $xy = e - e^y$, then $\left(\frac{d^2y}{dx^2}\right)_{x=0}$ is

23. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to

24. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

25. If $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$, then $P(n)$ is true for

26. If $n(U) = 700$, $n(A) = 200$, $n(B) = 240$, $n(A \cap B) = 100$, then $n(A^C \cup B^C)$ is equal to

27. If α, β are roots of the equation $p(x^2 - x) + x + 5 = 0$ and p_1, p_2 are two values of p for which the roots α, β are connected by the relation $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{5}$ then the value of $\frac{p_1}{p_2}+\frac{p_2}{p_1}$ equals

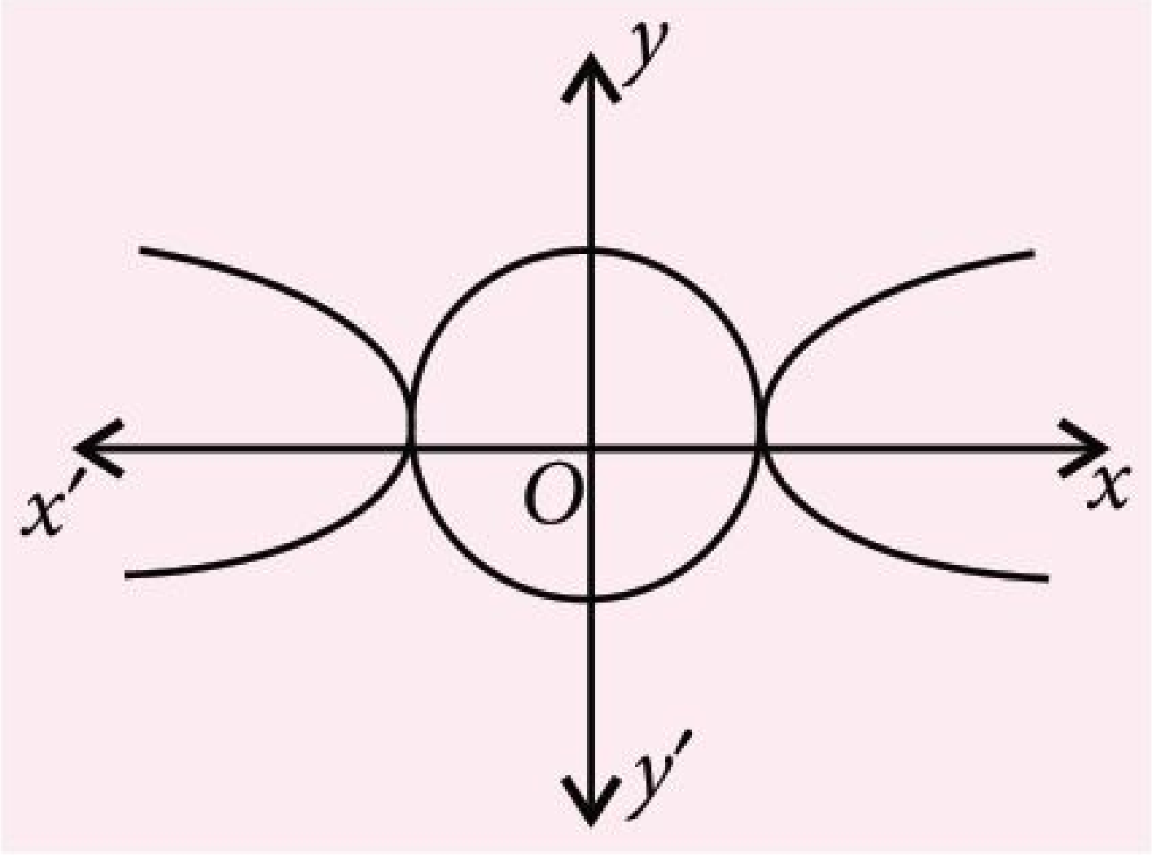
28. The third term of a G.P. is 7, the product of its first five terms is

29. If the coefficients of the r^{th} term and $(r + 1)^{\text{th}}$ term in the expansion of $(1 + x)^{20}$ are in the ratio 1 : 2, then r is equal to

30. If $\log_2 x + \log_2 y \geq 6$, then the least value of $x + y$ is

SOLUTIONS

1. (a) : For the circle $x^2 + y^2 = 1$ and rectangular hyperbola $x^2 - y^2 = 1$, one common tangent is evidently $x = 1$, the other being $x = -1$.
Since, $x = 1$ is nearer to the point $S(1/2, 1)$. Therefore we require the standard form of ellipse with focus at $S\left(\frac{1}{2}, 1\right)$ and directrix $x = 1$ which is



$$\left(x-\frac{1}{2}\right)^2+(y-1)^2=\left(\frac{1}{2}\right)^2(1-x)^2$$

$$\Rightarrow x^2-x+\frac{1}{4}+(y-1)^2=\frac{1}{4}(x^2-2x+1)$$

$$\Rightarrow \frac{3}{4}x^2-\frac{x}{2}+(y-1)^2=0$$

$$\Rightarrow \frac{3}{4}\left(x-\frac{1}{3}\right)^2+(y-1)^2=\frac{1}{12}$$

$$\Rightarrow 9\left(x-\frac{1}{3}\right)^2+12(y-1)^2=1$$

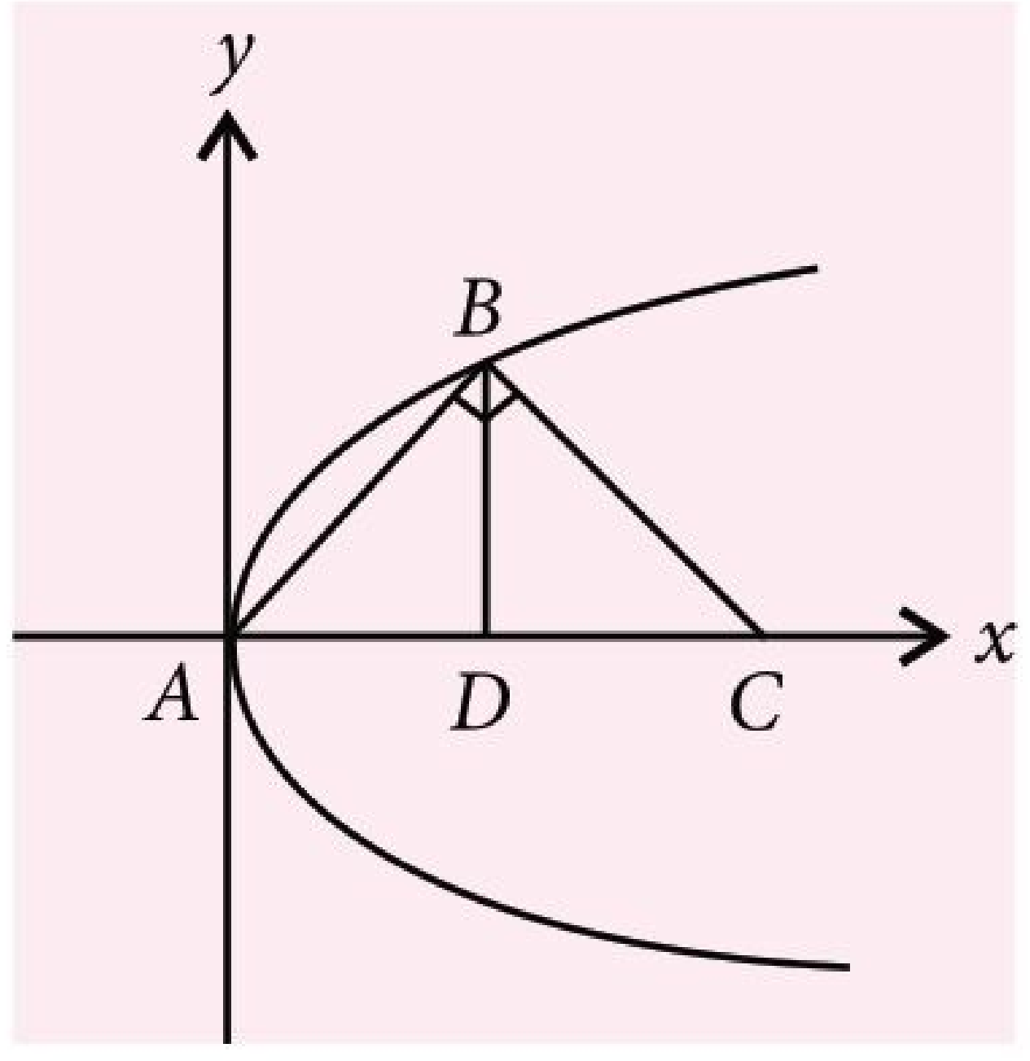
2. (c) : Let B be $(at^2, 2at)$

Slope of $AB = \frac{2at}{at^2}=\frac{2}{t}$

Equation of BC is

$$y-2at=-\frac{t}{2}(x-at^2)$$

$[\because BC \perp AB]$



Since BC meets x-axis at C.

∴ Ordinate of point C is zero.

$$0-2at=-\frac{t}{2}x+\frac{at^3}{2}$$

$4at = tx - at^3 \Rightarrow tx = 4at + at^3$

∴ $x = 4a + at^2$

Thus, coordinate of C is $(4a + at^2, 0)$ and D is $(at^2, 0)$.

∴ $DC = 4a + at^2 - at^2 = 4a$

3. (a) : The given equation is

$x^2 - 2ax + a^2 - 1 = 0$

$\Rightarrow x^2 - (a + 1)x - (a - 1)x + (a + 1)(a - 1) = 0$

$\Rightarrow x\{x - (a + 1)\} - (a - 1)\{x - (a + 1)\} = 0$

$\Rightarrow (x - a + 1)(x - a - 1) = 0$

$\Rightarrow x = a - 1$ or $x = a + 1$

Since, both the roots lie in $(-2, 1)$

∴ $-2 < a - 1 < 1$ and $-2 < a + 1 < 1$

$\Rightarrow -1 < a < 2$ and $-3 < a < 0$

∴ $-1 < a < 0 \Rightarrow [a] = -1$

4. (b) : Given $R = \{(x, y) : 2x + y = 8; x, y \in N\}$

Here $y = 8 - 2x$

∴ When $x = 1, y = 6 \in N$

$x = 2, y = 4 \in N$

$x = 3, y = 2 \in N$

$x = 4, y = 0 \notin N$

∴ $R = \{(1, 6), (2, 4), (3, 2)\}$

∴ Range of $R = \{y : (x, y) \in R\} = \{2, 4, 6\}$

5. (c) : Here $f(x)=\frac{\sin 4\pi[x]}{1+[x]^2}$

$f(x) = 0$ [since sine of integral multiple of π is 0]

∴ $f'(x) = 0 \ \forall \ x$

6. (d) : $|z|$ is a positive real number.

∴ $\text{Arg}(-|z|) = \pi$

∴ $\int_0^{100} \pi \, dx = 100\pi$

7. (b) : $\frac{1}{n!}\left[\frac{n!}{1!(n-1)!}+\frac{n!}{3!(n-3)!}+\frac{n!}{5!(n-5)!}+....\right]$

$$=\frac{1}{n!}[{}^nC_1+{}^nC_3+{}^nC_5+....+{}^nC_{n-1}]=\frac{1}{n!}\cdot 2^{n-1}$$

[∵ sum of the odd binomial coefficients is 2^{n-1}]

8. (c) : There should be exactly 4 correct and 4 wrong answers.

Let C stands for correct and W stands for wrong answers. One order of doing so is illustrated.

CCCCWWWW

The number of permutation of above mentioned demonstration is $\frac{8!}{4!\,4!}$.

Since each correct answer can be marked in 1 way and that for wrong in 3 ways.

$\therefore$ Required number of ways $=\frac{8!}{4!\,4!}\times 3^4=5670$

9. (b) : Let us mix the toys making $m + n$ number of distinct toys in all. Now Rina can pick up m toys out of these $m + n$ toys in ${}^{m+n}C_m$ ways which includes one way in which she can pick up her own original toys.

$\therefore$ Required number of ways $= {}^{m+n}C_m - 1$.

10. (c) : Let the roots are α and β

$\therefore$ $\alpha + \beta = -2a$ and $\alpha\beta = b$

Also given that $|\alpha - \beta| \le 2m$

$\Rightarrow (\alpha - \beta)^2 \le 4m^2$

Now, $(\alpha + \beta)^2 - 4\alpha\beta = (\alpha - \beta)^2$

$\Rightarrow 4a^2 - 4b \le 4m^2 \Rightarrow a^2 - b \le m^2$

$\Rightarrow a^2 - m^2 \le b$ ...(i)

Also, the discriminant $4a^2 - 4b \ge 0$ (as roots are real)

$\Rightarrow a^2 \ge b$ ...(ii)

From (i) and (ii), we have

$a^2 - m^2 \le b \le a^2$

i.e., $b \in [a^2 - m^2, a^2]$

11. (d) : We know that probability of any event lies between 0 and 1 and both 0 and 1 are inclusive.

$\therefore\ 0 \le \frac{1+3p}{4} \le 1$ gives $-\frac{1}{3} \le p \le 1$

$0 \le \frac{1-p}{3} \le 1$ gives $-2 \le p \le 1$

$0 \le \frac{1-3p}{2} \le 1$ gives $-\frac{1}{3} \le p \le \frac{1}{3}$

Since the events are mutually exclusive.

$\therefore\ 0 \le \frac{1+3p}{4}+\frac{1-p}{3}+\frac{1-3p}{2} \le 1$ gives $\frac{1}{13} \le p \le 1$

The set of all values of p is $\frac{1}{13} \le p \le \frac{1}{3}$.

12. (a) : The given function

$$f(x)=\begin{cases}3[x]-5\dfrac{|x|}{x}, & x\neq 0\\ 2, & x=0\end{cases}=\begin{cases}3[x]-5 & \text{when } x>0\\ 3[x]+5 & \text{when } x<0\\ 2 & \text{when } x=0\end{cases}$$

Now, $\int_{-3/2}^{2} f(x)dx$

$$=\int_{-3/2}^{-1} f(x)dx+\int_{-1}^{0} f(x)dx+\int_{0}^{1} f(x)dx+\int_{1}^{2} f(x)dx$$

$$=\int_{-3/2}^{-1} -1dx+\int_{-1}^{0} 2dx+\int_{0}^{1}(-5)dx+\int_{1}^{2}(-2)dx$$

$$=-1\left(-1+\frac{3}{2}\right)+2(0+1)+(-5)(1-0)+(-2)(2-1)$$

$$=-1\left(\frac{1}{2}\right)+2-5-2=-\frac{1}{2}+2-5-2=-\frac{11}{2}$$

13. (c)

14. (d) : Let $y=f(x) \Rightarrow x=f^{-1}(y)$ ...(i)

$\therefore\ y = 4x + 3 \Rightarrow x=\frac{y-3}{4}$

$\Rightarrow f^{-1}(y)=\frac{y-3}{4}$ [By using (i)]

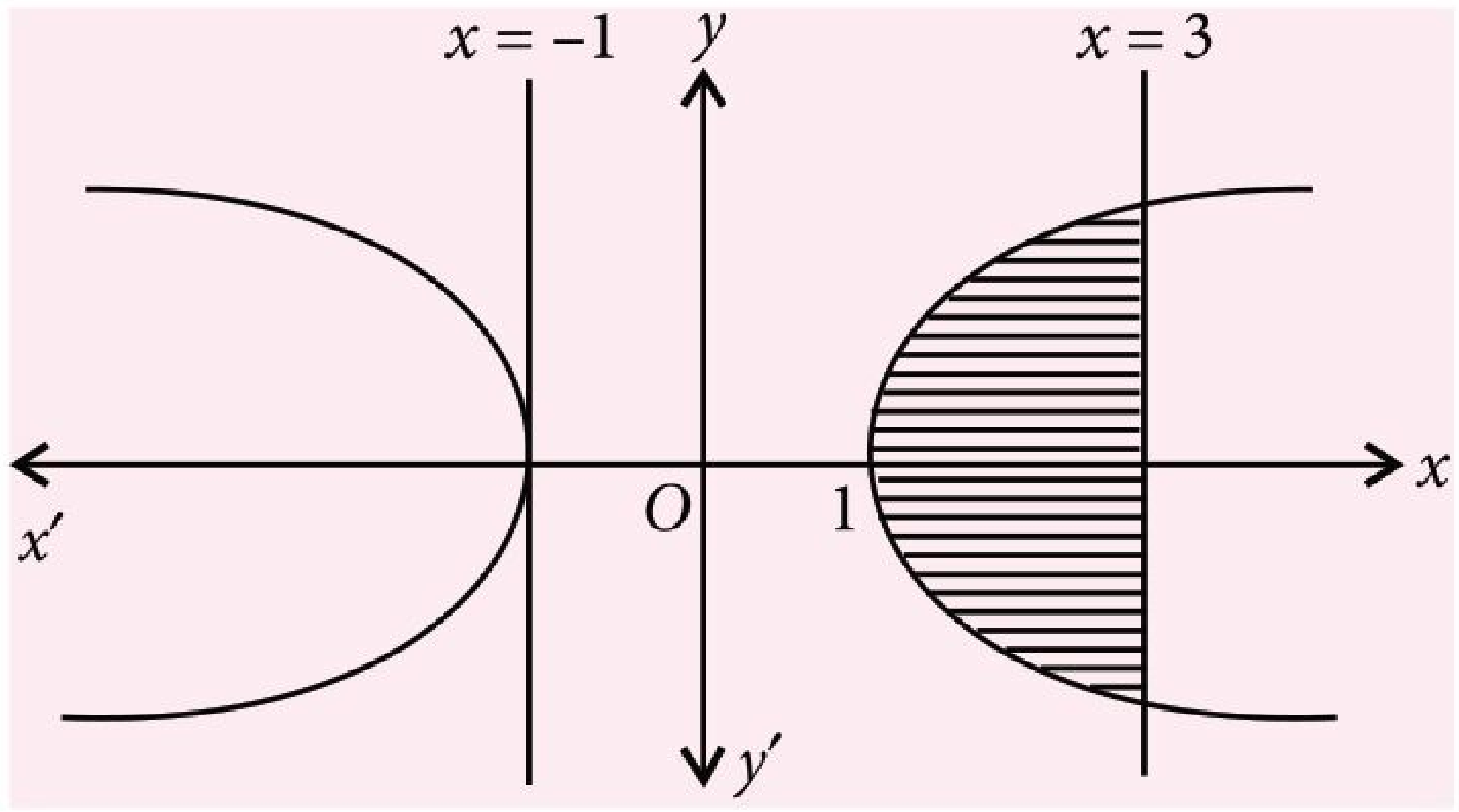
$\therefore\ g(y)=\frac{y-3}{4}$

15. (c) : The given equation of the curves

$|x - 1| \le 2$ and $x^2 - y^2 = 1$

$\therefore$ The graph of required curve is

$-1 \le x \le 3,\ x^2 - y^2 = 1$



$\therefore$ Required area $= 2\int_{1}^{3}\sqrt{x^2-1}\,dx$

$$=2\left[\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\log\left|x+\sqrt{x^2-1}\right|\right]_1^3$$

$$=\left[x\sqrt{x^2-1}-\log\left|x+\sqrt{x^2-1}\right|\right]_1^3$$

$$=6\sqrt{2}-\log|3+2\sqrt{2}|$$

16. (d) : Let $P(r)$ be the probability that the number r is drawn in one draw. It is given that

$P(r) = kr$, where k is a constant.

Further, $P(1) + P(2) + ... + P(2n) = 1$

$\Rightarrow\ k(1 + 2 + 3 + ... + 2n) = 1$

$$\Rightarrow\ k = \frac{1}{n(2n+1)}$$

Hence, the required probability

$= P(2) + P(4) + P(6) + ... + P(2n)$

$= 2k(1 + 2 + ... + n)$

$$= \frac{2}{n(2n+1)} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2n+1}$$

17. (a) : We know that $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{P(A \cap B)}{P(A)} \cdot \frac{P(A)}{P(B)}$$

$$P(A \mid B) = P(B \mid A) \cdot \frac{P(A)}{P(B)}$$

$$\therefore\ P(B) = \frac{\frac{2}{3} \times \frac{1}{4}}{\frac{1}{2}} = \frac{1}{3}$$

$$\left[\because\ P(B \mid A) = \frac{2}{3},\ P(A) = \frac{1}{4} \text{ and } P\left(A \mid B\right) = \frac{1}{2}\right]$$

18. (c) : Let $|A| \neq 0$

$\therefore\ A^{-1}$ exists.

Given that $AB = AC$

$\therefore\ A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow\ (A^{-1}A)B = (A^{-1}A)C$ [Associative law]

$\Rightarrow\ B = C$ [$\because\ AA^{-1} = \mathrm{I}$]

$\Rightarrow\ A$ is non-singular

$$\textbf{19. (a) :} \text{ Here } f(x) = \begin{vmatrix} \sin x & 1 & 0 \\ 1 & 2\sin x & 1 \\ 0 & 1 & 2\sin x \end{vmatrix}$$

$= \sin x(4\sin^2 x - 1) - 1(2\sin x)$

$\Rightarrow\ f(x) = 4\sin^3 x - 3\sin x$

$\therefore\ f(-x) = -4\sin^3 x + 3\sin x = -(4\sin^3 x - 3\sin x)$

$\Rightarrow\ f(-x) = -f(x)$

$\therefore\ f(x)$ is an odd function.

$$\therefore\ \int_{-\pi/2}^{\pi/2} f(x)dx = 0$$

$$\textbf{20. (b) :}\ f(x) = \frac{\sin x \cos x}{1 + \sin x + \cos x} = \frac{\sin x}{\sec x + \tan x + 1}$$

$$= \frac{\sin x}{(1 + \tan x) + \sec x} = \frac{\sin x(1 + \tan x - \sec x)}{(1 + \tan x)^2 - \sec^2 x}$$

$$= \frac{\cos x(1 + \tan x - \sec x)}{2} = \frac{\sin x + \cos x - 1}{2}$$

$$\textbf{21. }\ f(x) = e^{\cos^{-1}\sin\left(x + \frac{\pi}{3}\right)}$$

$$\therefore\ f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\sin\left(\frac{8\pi}{9} + \frac{\pi}{3}\right)}$$

$$= e^{\cos^{-1}\sin\left(\frac{11\pi}{9}\right)} = e^{\cos^{-1}\sin\left(\frac{22\pi}{18}\right)}$$

$$= e^{\cos^{-1}\sin\left(\frac{9\pi}{18} + \frac{13\pi}{18}\right)} = e^{\cos^{-1}\sin\left(\frac{\pi}{2} + \frac{13\pi}{18}\right)}$$

$$= e^{\cos^{-1}\cos\left(\frac{13\pi}{18}\right)} = e^{\frac{13\pi}{18}}$$

22. $xy = e - e^y$

$$\Rightarrow\ y + x\frac{dy}{dx} = -e^y\frac{dy}{dx}$$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

	7	9					6	5				1	7							3
4	5				7								9					8		
				8			1						2							
	8		7	2	5	3								1			6		3	
		5		6												1			2	8
2			1										3		2		7			4
6							7	2		8					7		1	5		
7		2			3	6	4				3									6
								1	7					2	5	4		9	1	
									1		2		5							
						4							9	1						
								6		4			3							
1	5					7				9				4		6	7		8	
	9			6	4		2					6							7	4
		3	5				9		2		6				4				6	2
					8	3								1	3				9	
			1	5	9	2								6		8				
5								1				4	3		9	1				
8				2	3							5	1							
2			7		6								4							6
	3		9			6		2						7	8		2	3		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$$\Rightarrow \quad e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0 \qquad \text{...(i)}$$

$$\therefore \quad e\frac{dy}{dx}+1=0 \qquad [\text{As } x=0 \quad \therefore \; y=1]$$

$$\Rightarrow \quad \frac{dy}{dx}=-\frac{1}{e}$$

Differentiating (i) with respect to x, we get

$$e^y\frac{d^2y}{dx^2}+e^y\left(\frac{dy}{dx}\right)^2+x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$$

Using $x = 0$, $y = 1$, $\frac{dy}{dx}=-\frac{1}{e}$, we get $\frac{d^2y}{dx^2}=\frac{1}{e^2}$

23. Let $\vec{a}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$

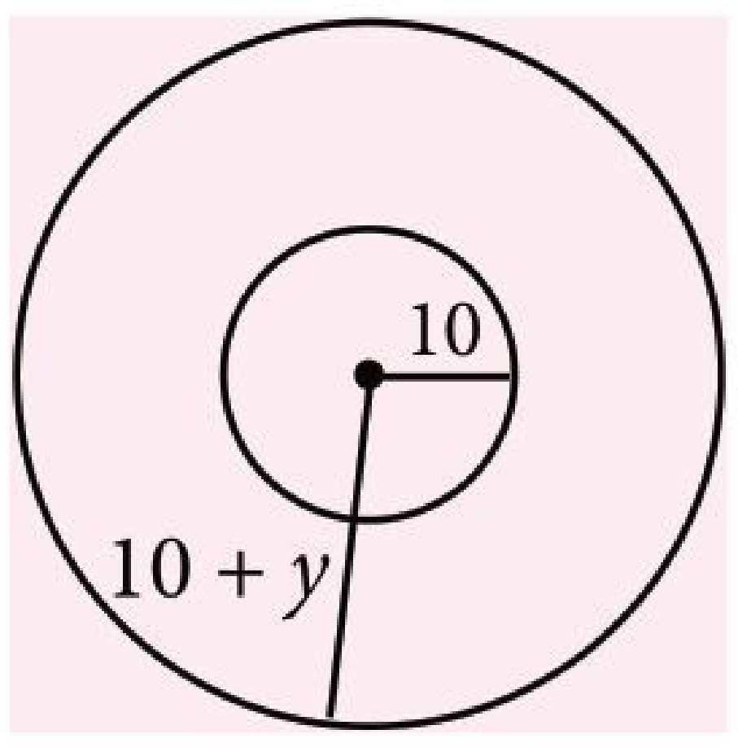
$$\therefore \; |\vec{a}|^2=a_1^2+b_1^2+c_1^2.$$

$$\therefore \; \vec{a}\times\hat{i}=-b_1\hat{k}+c_1\hat{j} \;\Rightarrow\; |\vec{a}\times\hat{i}|^2=b_1^2+c_1^2.$$

Similarly, $|\vec{a}\times\hat{j}|^2=a_1^2+c_1^2$ and $|\vec{a}\times\hat{k}|^2=a_1^2+b_1^2$

$$\therefore \; |\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2=2(a_1^2+b_1^2+c_1^2)=2|\vec{a}|^2$$

24. $V=\frac{4}{3}\pi(y+10)^3$, where y is thickness of ice



$$\frac{dV}{dt}=4\pi(10+y)^2\frac{dy}{dt}$$

$$\left(\frac{dy}{dt}\right)_{y=5}=\frac{50}{4\pi(15)^2} \qquad \left[\text{As } \frac{dV}{dt}=50 \text{ cm}^3/\text{min}\right]$$

$$=\frac{1}{18\pi} \text{ cm/min}$$

25. Let $P(n)=\frac{4^n}{n+1}<\frac{(2n)!}{(n!)^2}$

For $n = 1$, $P(n)$ is not true.

For $n = 2$, $P(2):\frac{4^2}{2+1}<\frac{4!}{(2)^2} \;\Rightarrow\; \frac{16}{3}<\frac{24}{4}$

which is true.

Let for $n = m > 2$, $P(n)$ is true *i.e.* $\frac{4^m}{m+1}<\frac{(2m)!}{(m!)^2}$

$$\text{Now, } \frac{4^{m+1}}{m+2}=\frac{4^m}{m+1}\cdot\frac{4(m+1)}{m+2}<\frac{(2m)!}{(m!)^2}\cdot\frac{4(m+1)}{m+2}$$

$$=\frac{(2m)!(2m+1)(2m+2)\times4(m+1)(m+1)^2}{(2m+1)(2m+2)(m!)^2(m+1)^2(m+2)}$$

$$=\frac{[2(m+1)]!}{[(m+1)!]^2}\cdot\frac{2(m+1)^2}{(2m+1)(m+2)}<\frac{[2(m+1)]!}{[(m+1)!]^2}$$

$$\left[\because \; \frac{2(m+1)^2}{(2m+1)(m+2)}<1 \;\forall m>2\right]$$

Hence, for $n \ge 2$, $P(n)$ is true.

26. We know, $n(A^C \cup B^C) = n[(A \cap B)^C]$

$= n(U) - n(A \cap B) = n(U) - 100 = 700 - 100 = 600$

27. $p(x^2 - x) + x + 5 = 0$

$\Rightarrow \; px^2 - (p - 1)x + 5 = 0$

$$\therefore \quad \alpha+\beta=\frac{p-1}{p} \text{ and } \alpha\beta=\frac{5}{p}$$

$$\text{Now, } \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{5} \;\Rightarrow\; \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{4}{5}$$

$$\Rightarrow \; \frac{(p-1)^2-10p}{5p}=\frac{4}{5} \;\Rightarrow\; p^2-16p+1=0$$

$\therefore \; p_1 + p_2 = 16$, $p_1p_2 = 1$

$$\text{Now, } \frac{p_1}{p_2}+\frac{p_2}{p_1}=\frac{(p_1+p_2)^2-2p_1p_2}{p_1p_2}$$

$\therefore$ Required value is 254.

28. Given that $ar^2 = 7 \quad \therefore \; a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4$

$= a^5 r^{1+2+3+4} = a^5 r^{10} = (ar^2)^5 = 7^5$

29. $t_r = t_{(r-1)+1} = {}^{20}C_{r-1}\, x^{r-1}$ and $t_{r+1} = {}^{20}C_r\, x^r$

According to the question,

$$\frac{{}^{20}C_{r-1}}{{}^{20}C_r}=\frac{1}{2} \;\Rightarrow\; \frac{r}{20-r+1}=\frac{1}{2}$$

$\Rightarrow \; 2r = 20 - r + 1 \Rightarrow 3r = 21$

$\therefore \; r = 7$

30. Given that $\log_2 x + \log_2 y \ge 6$

$\Rightarrow \; \log_2(xy) \ge 6 \Rightarrow xy \ge 64$

Also for $\log_2 x$ and $\log_2 y$ to be defined, $x > 0$, $y > 0$

$\because \;$ AM $\ge$ GM,

$$\therefore \; \frac{x+y}{2}\ge\sqrt{xy} \;\Rightarrow\; x+y\ge2\sqrt{xy}\ge2\sqrt{64}=16$$

◆◆

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(d)
6.	(b)	**7.**	(a, b, c)	**8.**	(a, c, d)	**9.**	(a, b)	**10.**	(a, d)
11.	(a, b, c)	**12.**	(a, b)	**13.**	(a, b, c)	**14.**	(a)	**15.**	(b)
16.	(b)	**17.**	(1)	**18.**	(2)	**19.**	(5)	**20.**	(5)

GEAR UP for JEE MAIN 2022

Exam Dates
Session-1
23rd to 29th June
Session-2
21st to 30th July

Section A will be of Multiple Choice Questions (MCQs). Section B will contain questions whose answers are to be filled in as a Numerical Value. In Section B candidates have to attempt any five questions out of 10.

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The set of values of b for which $f(x)$ has greatest value at $x = 1$, where $f(x)=\begin{cases} x^3-x^2+10x-7, & x<1 \\ -2x+\log_2(b^2-4), & x>1 \end{cases}$ is

(a) $(-6, -2)$ (b) $(2, 6)$
(c) $(-6, -2) \cup (2, 6)$ (d) $(-6, 6)$

2. Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(x) = ax + 3\sin x + 4\cos x$. Then $f(x)$ is invertible, if

(a) $a \in (-5, 5)$ (b) $a \in (-\infty, -5)$
(c) $a \in (5, \infty)$ (d) Both (b) and (c)

3. If $A=\begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then $(A(\text{adj}A)A^{-1})A =$

(a) $2\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1/6 & -1/6 \\ 2/6 & 1/6 & 3/6 \\ 3/6 & 2/6 & 1/6 \end{bmatrix}$ (d) none of these

4. Let p be an odd prime number and T_p be the following set of 2×2 matrices.

$$T_p=\left\{A=\begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, (p-1)\}\right\}$$

The number of matrices A in T_p such that A is either symmetric or skew symmetric or both and det(A) is divisible by p is

(a) $(p - 1)^2$ (b) $2(p - 1)$
(c) $(p - 1)^2 + 1$ (d) $2p - 1$

5. If $A = \{\theta : 2\cos^2\theta + \sin\theta \le 2\}$ and $B=\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right\}$, then $A \cap B =$

(a) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\}$

(b) $\left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$

(c) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \cup \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$

(d) none of these

6. The relation R defined on a set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 16\}$ is given by

(a) $R = \{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$
(b) $R = \{(2, 2), (3, 2), (4, 2), (2, 4)\}$
(c) $R = \{(3, 3), (4, 3), (5, 4), (3, 4)\}$
(d) none of these

7. If $f : A \to B$ given by $3^{f(x)} + 2^{-x} = 4$ is a bijection, then

(a) $A = \{x \in \mathbf{R} : -1 < x < \infty\}$, $B = \{x \in \mathbf{R} : 2 < x < 4\}$
(b) $A = \{x \in \mathbf{R} : -3 < x < \infty\}$, $B = \{x \in \mathbf{R} : 0 < x < 4\}$
(c) $A = \{x \in \mathbf{R} : -2 < x < \infty\}$, $B = \{x \in \mathbf{R} : 0 < x < 4\}$
(d) none of these

8. If 3 arithmetic means, 3 geometric means and 3 harmonic means are inserted between 1 and 5, then the cubic equation whose roots are first A.M., second G.M. and third H.M. between 1 and 5, is

(a) $x^3-\left(\frac{9}{2}+\sqrt{5}\right)x^2+\left(\frac{9\sqrt{5}}{2}+5\right)x-5\sqrt{5}=0$

(b) $x^3+\left(\frac{9}{2}+\sqrt{5}\right)x^2-\left(\frac{9\sqrt{5}}{2}+5\right)x-5\sqrt{5}=0$

(c) $x^3+\left(\frac{9}{2}-\sqrt{5}\right)x^2-\left(\frac{9\sqrt{5}}{2}-5\right)x+5\sqrt{5}=0$

(d) none of these

9. A circle circumscribing an equilateral triangle with centroid at (0, 0) of side 'a' is drawn and a square is drawn whose four sides touch the circle. The equation of the circle circumscribing the square is

(a) $x^2 + y^2 = 2a^2$ (b) $3x^2 + 3y^2 = 2a^2$
(c) $5x^2 + 5y^2 = 3a^2$ (d) none of these

10. Let X and Y be two events such that $P(X/Y) = \frac{1}{2}, P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is incorrect?

(a) $P(X \cup Y) = \frac{2}{3}$
(b) X and Y are independent events
(c) $P(\bar{X} \cap Y) = \frac{1}{6}$
(d) X and Y are not independent events

11. The mean deviation and standard deviation about actual mean of the series $a, a + d, a + 2d, ..., a + 2nd$ are respectively

(a) $\frac{n(n+1)d}{2n+1}, \sqrt{\frac{n(n-1)}{3}}\cdot d$ (b) $\frac{n(n-1)}{3}, \frac{n(n+1)}{2n}\cdot d$
(c) $\frac{n(n+1)d}{2n+1}, \sqrt{\frac{n(n+1)}{3}}\cdot d$ (d) $\frac{n(n-1)d}{2n-1}, \sqrt{\frac{n(n-1)}{3}}\cdot d$

12. If the sum of the coefficients in the expansion of $(b + c)^{20}\,[1 + (a - 2)x]^{20}$ is equal to square of the sum of the coefficients in the expansion of $[2bcx - (b + c)y]^{10}$, where a, b, c are positive constants, then

(a) $a \geq \sqrt{bc}$ (b) $\frac{b+c}{2} \geq a$
(c) $\frac{1}{c}, \frac{1}{a}, \frac{1}{b}$ are in G.P. (d) none of these

13. Area bounded by $|x - 1| \leq 2$ and $x^2 - y^2 = 1$, is

(a) $6\sqrt{2} + \frac{1}{2}\log|3 + 2\sqrt{2}|$ (b) $6\sqrt{2} + \frac{1}{2}\log|3 - 2\sqrt{2}|$
(c) $6\sqrt{2} - \log|3 + 2\sqrt{2}|$ (d) none of these

14. If in a ΔABC, sin$A$, sin$B$ and sin$C$ are in A.P., then
(a) the altitudes are in A.P.
(b) the altitudes are in H.P.
(c) the medians are in G.P.
(d) the medians are in A.P.

15. Let $\vec{A}$ be a vector parallel to line of intersection of planes P_1 and P_2 through the origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$, P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is

(a) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ or $\frac{3\pi}{2}$
(c) $\frac{\pi}{6}$ or $\frac{\pi}{3}$ (d) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

16. Let S and S' be two foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If a circle described on SS' as diameter intersects the ellipse in real and distinct points, then the eccentricity e of the ellipse satisfies

(a) $e = 1/\sqrt{2}$ (b) $e \in (1/\sqrt{2}, 1)$
(c) $e \in (0, 1/\sqrt{2})$ (d) none of these

17. A family of lines given by $(1 + 2\lambda)x + (1 - \lambda)y + \lambda = 0$, λ being the parameter. The line belonging to this family at the maximum distance from the point (1, 4), is

(a) $4x - y + 1 = 0$ (b) $33x + 12y + 7 = 0$
(c) $12x + 33y = 7$ (d) none of these

18. If a particle moving along a line follows the law $s = \sqrt{1+t}$, then the acceleration is proportional to
(a) square of the velocity
(b) cube of the displacement
(c) cube of the velocity
(d) square of the displacement

19. $\int \sin(101x)\sin^{99} x\, dx$ equals

(a) $\frac{1}{100}\sin(100x)(\sin x)^{100} + C$
(b) $\frac{1}{100}\cos(100x)(\sin x)^{100} + C$
(c) $\frac{1}{100}\cos(100x)(\cos x)^{100} + C$
(d) $\frac{1}{100}\sin(100x)(\sin x)^{101} + C$

20. Tangent is drawn at any point P of the curve which passes through (1, 1) cutting x-axis and y-axis at A and B respectively. If $AP : PB = 3 : 1$, then

(a) the differential equation of the curve is $3x\frac{dy}{dx} + y = 0$ and the curve passes through (1/8, 2)
(b) the differential equation of the curve is $3x\frac{dy}{dx} - y = 0$ and the curve passes through (1/8, –2)
(c) the curve is passing through (–1/8, 2)
(d) the normal at (1, 1) is $x + 3y = 4$

SECTION-B (NUMERICAL VALUE TYPE)

21. Between two junction stations A and B there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive, is

22. If $\left|\frac{z-6}{z-8}\right|=1$, then the value of $x \in \mathbf{R}$, where

$$z=x+i\begin{vmatrix} -3 & 2i & 2+i \\ -2i & 2 & 4-3i \\ 2-i & 4+3i & 7 \end{vmatrix}, \text{ is}$$

23. If 1 lies between the roots of the equation $y^2 - my + 1 = 0$ and $[x]$ denotes the greatest integer less than or equal to x then the value of $\left[\left(\frac{4|x|}{|x|^2+16}\right)^m\right]$ is

24. The value of sec40° + sec80° + sec160° will be

25. The value of $\tan\{\sin^{-1}(\cos(\sin^{-1}x))\}$

$\tan\{\cos^{-1}(\sin(\cos^{-1}x))\}$, where $0<x<\frac{\pi}{2}$ is equal to

26. The value of 'a' so that the volume of the parallelepiped formed by $\hat{i}+a\hat{j}+\hat{k},\ \hat{j}+a\hat{k},\ a\hat{i}+\hat{k}$ becomes minimum is

27. If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{a}\cdot\vec{b}=1,\ \vec{a}\times\vec{b}=\hat{j}-\hat{k}$ and $\vec{b}=l\hat{i}+m\hat{j}+n\hat{k}$, then $l+m+n=$

28. If a curve is represented parametrically by the equations : $x = 4t^3 + 3$ and $y = 4 + 3t^4$ and $\frac{d^2x}{dy^2}\Big/\left(\frac{dx}{dy}\right)^n$ is constant, then the value of n is

29. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then the sum of all possible values of $k =$

30. Let f be a function defined by $f(x)=\int_1^x t(t^2-3t+2)dt$, $1 \le x \le 3$. Then the range of $f(x)$ is $\left[\frac{-1}{2k}, k\right]$, where $k =$

SOLUTIONS

1. **(c) :** We have, $f'(x)=\begin{cases} 3x^2-2x+10, & x<1 \\ -2 & , x>1 \end{cases}$

Clearly, $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$

Thus, $f(x)$ is increasing for all $x < 1$ and decreasing for $x > 1$.

Therefore, $f(x)$ will have greatest value at $x = 1$,

if $\lim\limits_{x\to1^+} f(x)<f(1)$ or $\lim\limits_{x\to1^-} f(x)<f(1)$

$\Rightarrow \lim\limits_{x\to1} -2x+\log_2(b^2-4)<1-1+10-7$

$\Rightarrow -2 + \log_2(b^2 - 4) < 3 \Rightarrow \log_2(b^2 - 4) < 5$

$\Rightarrow b^2 - 4 < 2^5$ and $b^2 - 4 > 0$

$\Rightarrow b^2 - 36 < 0$ and $b^2 - 4 > 0$

$\Rightarrow -6 < b < 6$ and $b \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow b \in (-6, -2) \cup (2, 6)$

2. **(d) :** We have, $f(x) = ax + 3\sin x + 4\cos x$

$\Rightarrow f'(x) = a + 3\cos x - 4\sin x$

If $f(x)$ is invertible, then

$f'(x) > 0\ \forall\ x$ or $f'(x) < 0\ \forall\ x$

$\Rightarrow a + 3\cos x - 4\sin x > 0\ \forall\ x$

or $a + 3\cos x - 4\sin x < 0\ \forall\ x$

$\Rightarrow a - 5 > 0$ or $a + 5 < 0$ [$\because -5 \le 3\cos x - 4\sin x \le 5$]

$\Rightarrow a > 5$ or $a < -5 \Rightarrow a \in (-\infty, -5) \cup (5, \infty)$

3. **(a) :** We have, $|A|=\begin{vmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix}=0+7-1=6$

$\therefore (A(\text{adj}A)A^{-1})A = (A(\text{adj}A))(A^{-1}A) = (|A|I)I$

$$\Rightarrow (A(\text{adj}A)A^{-1})A=|A|I=\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}=2\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4. **(d) :** **Case-I :** When A is both symmetric as well as skew-symmetric.

In this case, we have $A = 0$ (Null matrix)

$\therefore |A| = 0 \Rightarrow |A|$ is divisible by p.

Case-II : When A is skew-symmetric matrix.

In this case, we have $a = 0$ and $c = -b$

$\therefore |A| = \begin{vmatrix} 0 & b \\ -b & 0 \end{vmatrix} = b^2$

Now, $p/|A| \Rightarrow p/b^2 \Rightarrow b = 0$

[$\because b \in \{0, 1, 2, 3, ..., (p - 1)\}$]

$\therefore A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Case-III : When A is symmetric matrix

In this case, we have $c = b$

$\therefore A = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \Rightarrow |A| = a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow p/(a - b)$ or $p/(a + b)$ [$\because p$ is prime]

When $p/a - b \Rightarrow a = b$. So, there are p choices for matrix A including $A = 0$.

When $p / a + b$

$\Rightarrow \quad a = r,\ b = p - r,$ where $r = 0, 1, 2, \ldots, p - 1.$

So, there are p choices for matrix A including $A = 0$.

Hence, there are $(2p - 1)$ choices.

5. (c) : We have, $2\cos^2\theta + \sin\theta \le 2$

$\Rightarrow \quad 2(1 - \sin^2\theta) + \sin\theta \le 2$

$\Rightarrow \quad 2\sin^2\theta - \sin\theta \ge 0$

$\Rightarrow \quad \sin\theta(2\sin\theta - 1) \ge 0$

Now, following two cases arises :

Case I: When $\sin\theta \ge 0$ and $(2\sin\theta - 1) \ge 0$

$$\Rightarrow \quad \sin\theta \ge 0 \text{ and } \sin\theta \ge \frac{1}{2} \quad \Rightarrow \quad \frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$$

$$\therefore \quad A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \left[\because B = \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right]$$

Case II : When $\sin\theta \le 0$ and $(2\sin\theta - 1) \le 0$

$$\Rightarrow \quad \sin\theta \le 0 \text{ and } \sin\theta \le \frac{1}{2}$$

$\Rightarrow \quad \sin\theta \le 0 \quad \Rightarrow \quad \pi \le \theta \le 2\pi$

$$\therefore \quad A \cap B = \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\} \left[\because B = \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right]$$

Thus, $A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \cup \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$

6. (d) : We have, $(a, b) \in R \Leftrightarrow |a^2 - b^2| < 16$

When $a = 1 \Rightarrow |1 - b^2| < 16 \Rightarrow |b^2 - 1| < 16$

$\Rightarrow \quad -15 < b^2 < 17 \Rightarrow 0 < b^2 < 17 \Rightarrow b = 1, 2, 3, 4$

When $a = 2 \Rightarrow |4 - b^2| < 16 \Rightarrow |b^2 - 4| < 16$

$\Rightarrow \quad -12 < b^2 < 20 \Rightarrow 0 < b^2 < 20 \Rightarrow b = 1, 2, 3, 4$

When $a = 3 \Rightarrow |9 - b^2| < 16 \Rightarrow |b^2 - 9| < 16$

$\Rightarrow \quad -7 < b^2 < 25 \Rightarrow 0 < b^2 < 25 \Rightarrow b = 1, 2, 3, 4$

When $a = 4 \Rightarrow |16 - b^2| < 16 \Rightarrow |b^2 - 16| < 16$

$\Rightarrow \quad 0 < b^2 < 32 \Rightarrow b = 1, 2, 3, 4, 5$

When $a = 5 \Rightarrow |25 - b^2| < 16 \Rightarrow |b^2 - 25| < 16$

$\Rightarrow \quad 9 < b^2 < 41 \Rightarrow b = 4, 5.$

$$\therefore \quad R = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4), \\ (3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4), \\ (4,5),(5,4),(5,5) \end{Bmatrix}$$

7. (d) : We have, $3^{f(x)} + 2^{-x} = 4$

$\Rightarrow \quad 3^{f(x)} = 4 - 2^{-x} \Rightarrow f(x) = \log_3(4 - 2^{-x})$

Clearly, $f(x)$ will be defined, if $4 - 2^{-x} > 0$

$\Rightarrow \quad 2^{-x} < 4 \Rightarrow 2^x > 2^{-2} \Rightarrow x > -2$

$\Rightarrow \quad A = \{x \in \mathbf{R} : x > -2\}$

Now, $f(x) = \log_3(4 - 2^{-x})$

$$\Rightarrow \quad f'(x) = \frac{2^{-x}\log 2}{(4 - 2^{-x})\log 3} > 0 \ \forall\, x > -2$$

$\Rightarrow \quad f(x)$ is increasing on $(-2, \infty)$

$\Rightarrow \quad$ Range of $f(x) = \{y : f(-2) < y < f(\infty)\}$

$\Rightarrow \quad B = \{y : -\infty < y < \log_3 4\} = \{x : -\infty < x < \log_3 4\}$

8. (a) : Let a = First A.M. between 1 and 5

$$\Rightarrow \quad a = 1 + \frac{5 - 1}{3 + 1} = 2$$

b = Second G.M. between 1 and 5

$$\Rightarrow \quad b = 1\left(\frac{5}{1}\right)^{2/4} = \sqrt{5}$$

c = Third H.M. between 1 and 5

$$\Rightarrow \quad c = \frac{1}{1 + 3 \times \left(\frac{1}{5} - 1\right) \times \frac{1}{4}} = \frac{5}{2}$$

$$\therefore \quad a + b + c = 2 + \sqrt{5} + \frac{5}{2},\ ab + bc + ca = 2\sqrt{5} + \frac{5\sqrt{5}}{2} + 5$$

and $abc = 5\sqrt{5}$

The cubic equation having a, b, c as its roots is

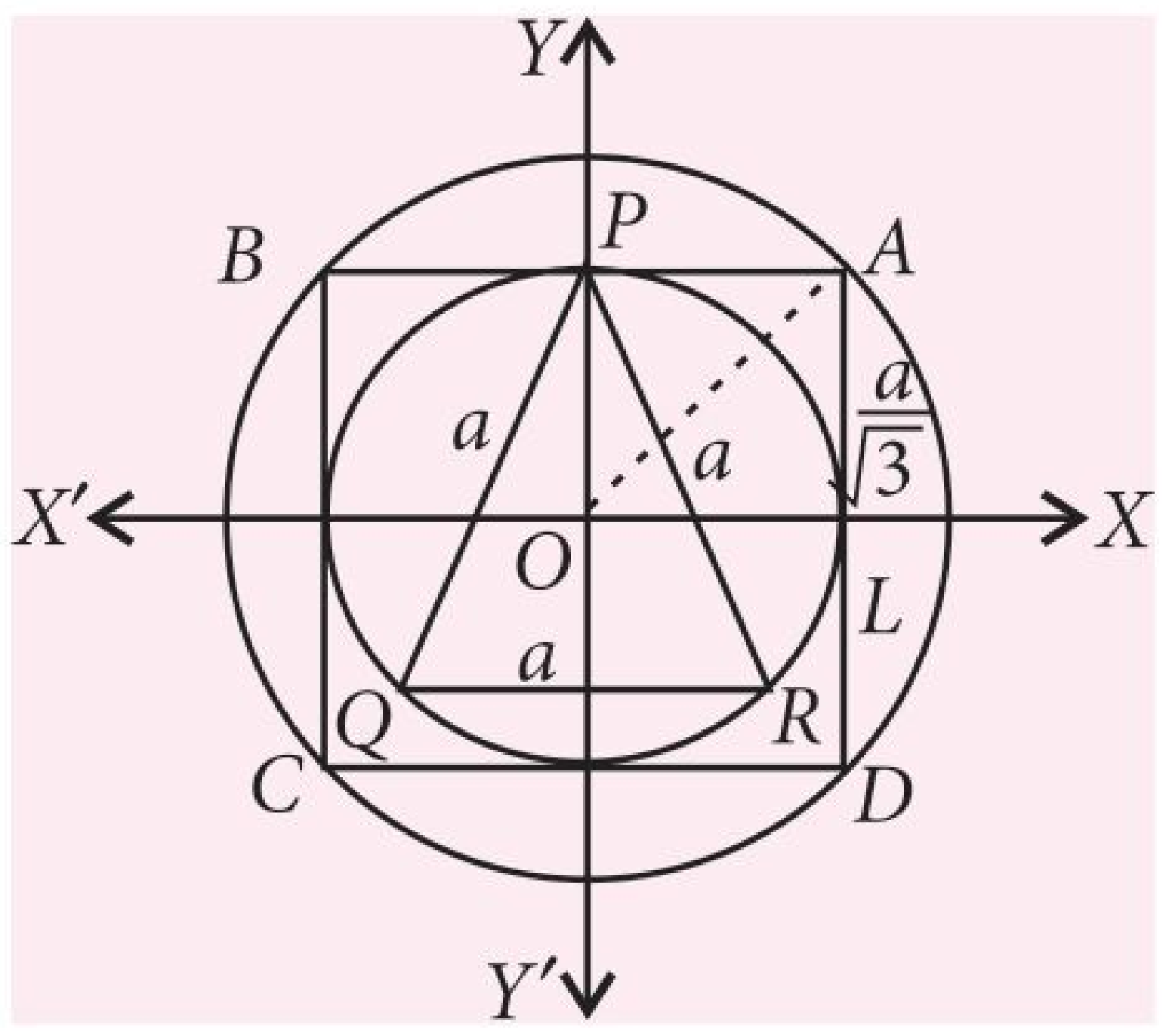
$$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc = 0$$

$$\Rightarrow \quad x^3 - \left(\frac{9}{2} + \sqrt{5}\right)x^2 + \left(\frac{9\sqrt{5}}{2} + 5\right)x - 5\sqrt{5} = 0.$$

9. (b) : Clearly, centre of the circumcircle is at origin and radius is two-third of the altitude $\frac{\sqrt{3}}{2}a.$

So, the equation of the circumcircle is

$$x^2 + y^2 = \left(\frac{a}{\sqrt{3}}\right)^2$$



Clearly the radius of the circumcircle of the square $ABCD$

$$= OA = \sqrt{OL^2 + AL^2}$$

$$= \sqrt{\frac{a^2}{3} + \frac{a^2}{3}} = \sqrt{\frac{2}{3}}a$$

So, the equation of the circumcircle is

$$x^2 + y^2 = \frac{2}{3}a^2 \Rightarrow 3(x^2 + y^2) = 2a^2$$

10. (d) : Given, $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$

$$\therefore \quad P(X) = \frac{P(X \cap Y)}{P(Y/X)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$$

Also, $P(Y)=\dfrac{P(X\cap Y)}{P(X/Y)}=\dfrac{\frac{1}{6}}{\frac{1}{2}}=\dfrac{1}{3}$

Clearly, $P(X\cap Y)=P(X)P(Y)$. So X and Y are independent events.

$$\therefore\quad P(X\cup Y)=1-P(\bar{X})P(\bar{Y})=1-\frac{1}{2}\times\frac{2}{3}=\frac{2}{3}.$$

11. (c) : Mean $\bar{x}=\dfrac{a+(a+d)+\ldots+(a+2nd)}{2n+1}=a+nd$

x_i	$\lvert x_i-\bar{x}\rvert$	$\lvert x_i-\bar{x}\rvert^2$
a	nd	n^2d^2
$a+d$	$(n-1)d$	$(n-1)^2d^2$
....	....	....
....	....	....
$a+(n-2)d$	$2d$	$4d^2$
$a+(n-1)d$	d	d^2
$a+nd$	0	0
$a+(n+1)d$	d	d^2
$a+(n+2)d$	$2d$	$4d^2$
....	....	....
....	....	....
$a+2nd$	nd	n^2d^2

$$\sum|x_i-\bar{x}|=2dn\left(\frac{n+1}{2}\right)=n(n+1)d,$$

$$\sum|x_i-\bar{x}|^2=2d^2[1^2+2^2+3^2.....+n^2]$$

$$=2d^2\,\frac{n(n+1)(2n+1)}{6}$$

$$\text{Now, M.D.}=\frac{1}{2n+1}\sum|x_i-\bar{x}|=\frac{n(n+1)d}{2n+1}$$

$$\text{S.D.}=\sqrt{\frac{1}{2n+1}\sum|x_i-\bar{x}|^2}=\sqrt{\frac{2d^2\cdot n(n+1)(2n+1)}{6(2n+1)}}$$

$$=\sqrt{\frac{n(n+1)d^2}{3}}=\sqrt{\frac{n(n+1)}{3}}\cdot d$$

12. (b) : The sum of the coefficients in the expansion of $(b+c)^{20}[1+(a-2)x]^{20}$ is given by

$S_1=(b+c)^{20}(a-1)^{20}$ [Putting $x=1$]

The sum of the coefficients in the expansion of $[2bcx-(b+c)y]^{10}$ is given by

$S_2=[2bc-(b+c)]^{10}$ [Putting $x=y=1$]

It is given that $S_1=S_2^{\,2}$

$\Rightarrow\quad (b+c)^{20}(a-1)^{20}=[2bc-(b+c)]^{20}$

$\Rightarrow\quad (b+c)(a-1)=[2bc-(b+c)]$

$\Rightarrow\ (a-1)=\dfrac{2bc}{b+c}-1\ \Rightarrow\quad a=\dfrac{2bc}{b+c}$

$\Rightarrow\quad a$ is the H.M. of b and c.

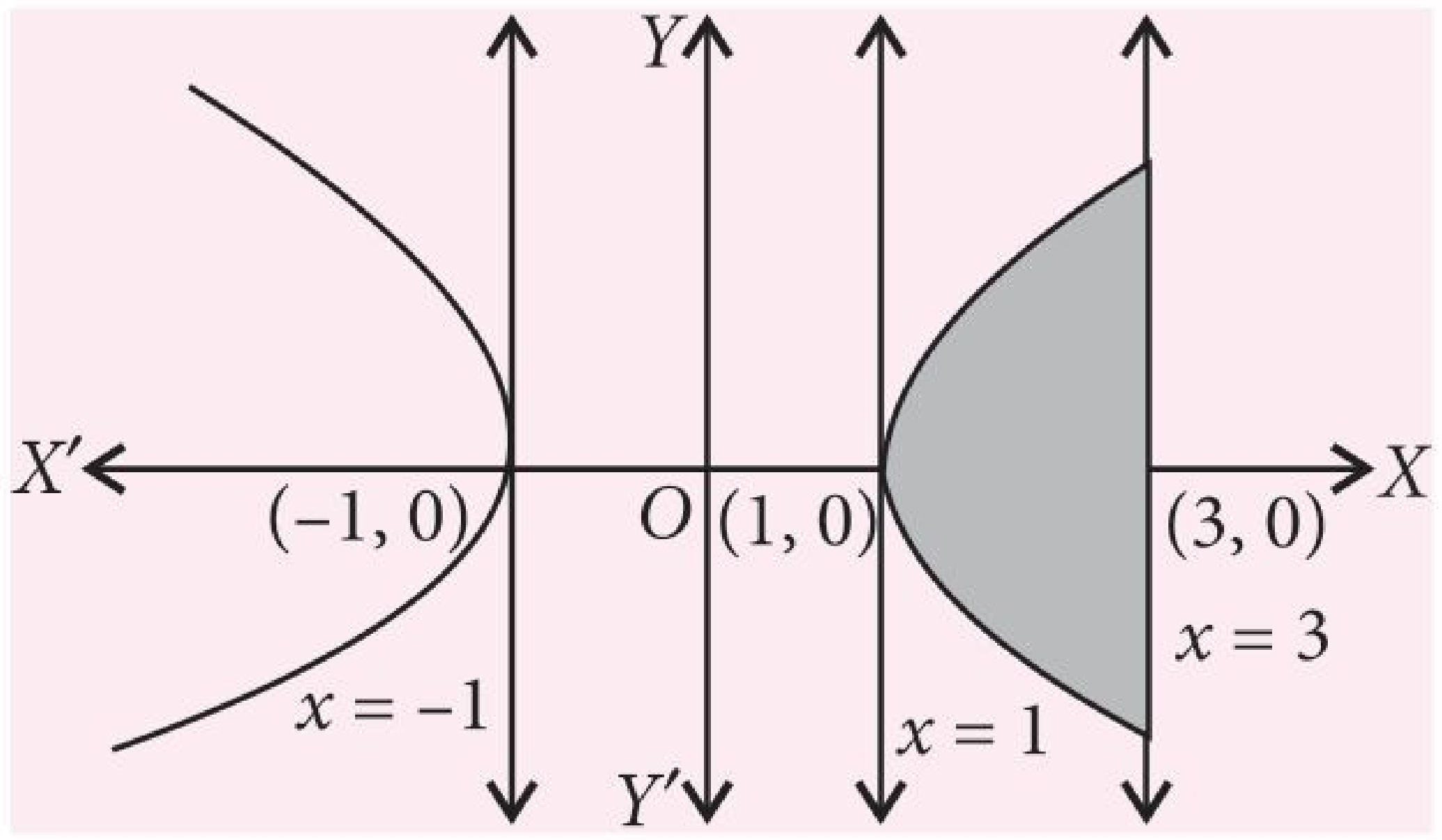
But, $\dfrac{b+c}{2}$ is the A.M. of b and c.

Using A.M. ≥ G.M. ≥ H.M., we get $\dfrac{b+c}{2}\geq a$.

13. (c) : We have, $|x-1|\leq 2\Leftrightarrow -1\leq x\leq 3$

∴ Required area A is the area of the shaded region and is given by $A=2\displaystyle\int_1^3\sqrt{x^2-1}\,dx$

$$\Rightarrow\ A=\left[x\sqrt{x^2-1}-\log_e(x+\sqrt{x^2-1})\right]_1^3$$

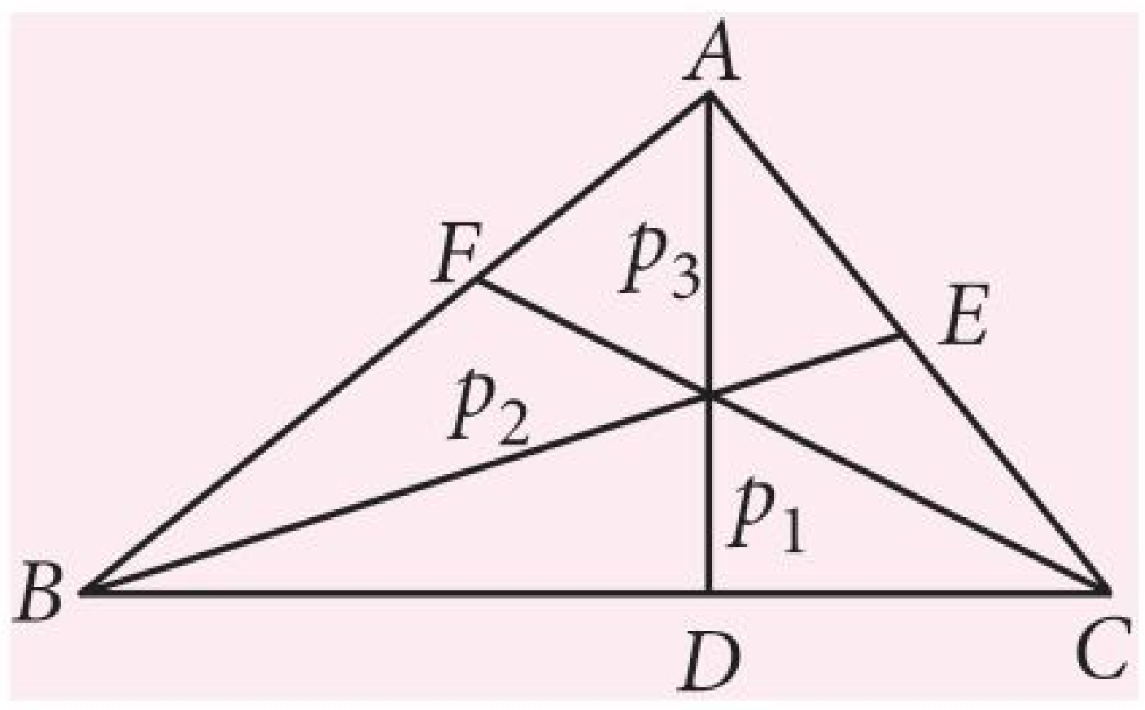


$$=\{3\sqrt{8}-\log_e(3+\sqrt{8})\}=6\sqrt{2}-\log_e(3+2\sqrt{2})$$

14. (b) : Let p_1, p_2, p_3 be the altitudes of ΔABC.

Then, $\Delta=\dfrac{1}{2}ap_1=\dfrac{1}{2}bp_2=\dfrac{1}{2}cp_3$

$\Rightarrow\ a=\dfrac{2\Delta}{p_1}, b=\dfrac{2\Delta}{p_2}, c=\dfrac{2\Delta}{p_3}$



Now, $\sin A$, $\sin B$, $\sin C$, are in A.P.

$\Rightarrow\quad a, b, c$ are in A.P.

$\Rightarrow\quad \dfrac{2\Delta}{p_1},\dfrac{2\Delta}{p_2},\dfrac{2\Delta}{p_3}$ are in A.P.

$\Rightarrow\quad \dfrac{1}{p_1},\dfrac{1}{p_2},\dfrac{1}{p_3}$ are in A.P. $\Rightarrow$ p_1, p_2, p_3 are in H.P.

15. (a) : Vectors parallel to the normals to P_1 and P_2 be $\vec{n}_1=(2\hat{j}+3\hat{k})\times(4\hat{j}-3\hat{k})$ and $\vec{n}_2=(\hat{j}-\hat{k})\times(3\hat{i}+3\hat{j})$ respectively. Clearly $\vec{n}_1=-18\hat{i}$ and $\vec{n}_2=3\hat{i}-3\hat{j}-3\hat{k}$

$\therefore\quad \vec{A}=\pm(\vec{n}_1\times\vec{n}_2)=\pm18\hat{i}\times(3\hat{i}-3\hat{j}-3\hat{k})=\pm54(\hat{j}-\hat{k})$

Let $\vec{B}=2\hat{i}+\hat{j}-2\hat{k}$

Let θ be the angle between $\vec{A}$ and $\vec{B}$.

Then, $\cos\theta=\dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}=\pm\dfrac{54(1+2)}{3\times54\sqrt{2}}=\pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow\quad \theta=\dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

16. (b) : The equation of the circle described on SS' as a diameter is

$(x - ae)(x + ae) + (y - 0)(y - 0) = 0$

$\Rightarrow \ x^2 + y^2 = a^2e^2$

The abscissa of the points of intersection of the ellipse and this circle are the roots of the equation

$$\frac{x^2}{a^2}+\frac{a^2e^2-x^2}{b^2}=1$$

$$\Rightarrow \ x^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)=1-\frac{a^2e^2}{b^2}$$

$$\Rightarrow \ x^2\left(\frac{b^2-a^2}{a^2b^2}\right)=\frac{b^2-a^2e^2}{b^2}$$

$$\Rightarrow \ -\frac{x^2a^2e^2}{a^2}=a^2-2a^2e^2$$

$$\Rightarrow \ x^2 = a^2\left(\frac{2e^2-1}{e^2}\right) \ \Rightarrow \ x = \pm\frac{a}{e}\sqrt{2e^2-1}$$

This will give distinct values of x if $2e^2 - 1 > 0$

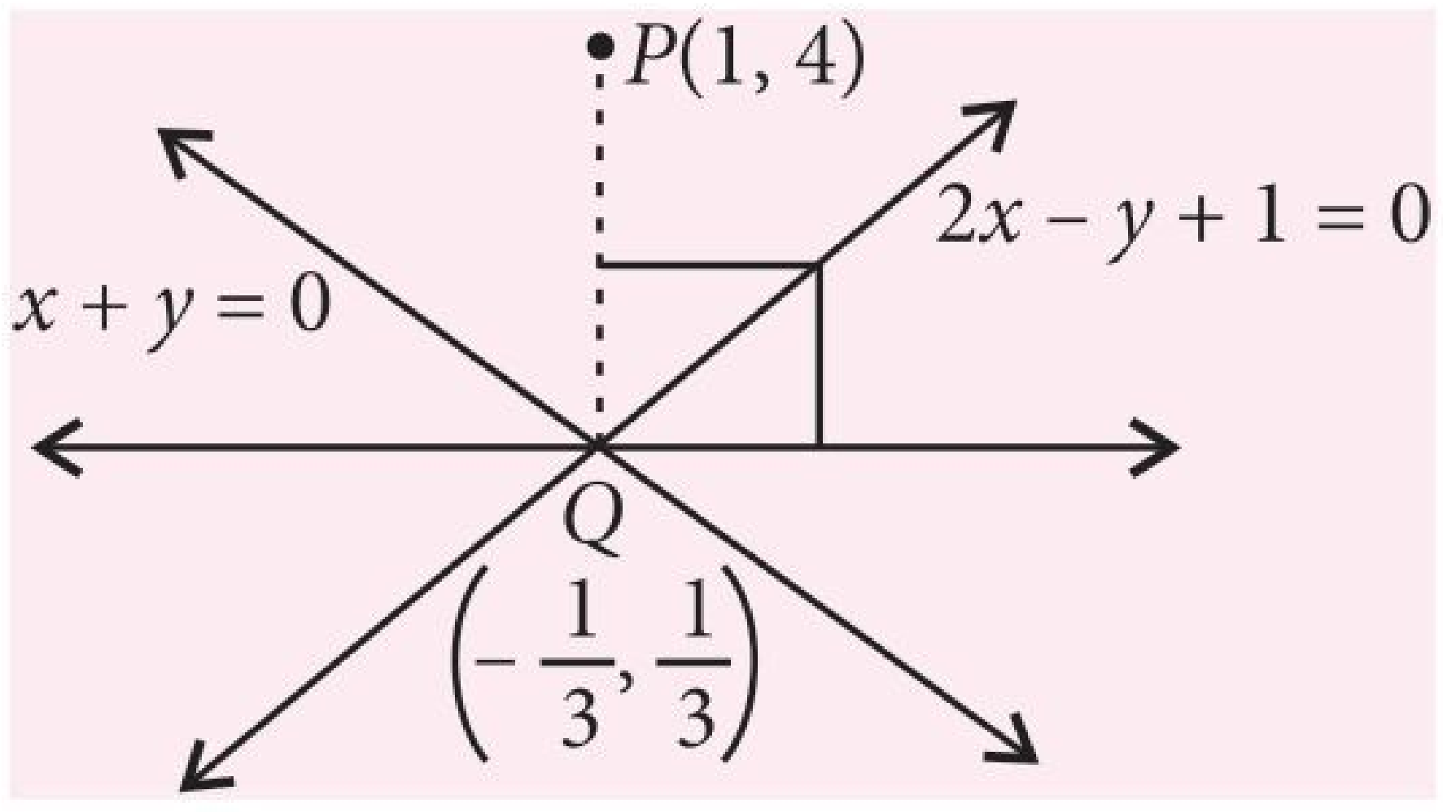
i.e. $e>\frac{1}{\sqrt{2}}$. Hence, $e\in\left(\frac{1}{\sqrt{2}},1\right)$

17. (c) : We have, $x + y + \lambda(2x - y + 1) = 0$ ...(i)

Clearly, (i) represents a family of lines passing through the point of intersection of the lines

$x + y = 0$ and $2x - y + 1 = 0$ *i.e.*, $\left(-\frac{1}{3},\frac{1}{3}\right)$

The required line passes through $\left(-\frac{1}{3},\frac{1}{3}\right)$ and is perpendicular to the line joining $(1, 4)$ and $\left(-\frac{1}{3},\frac{1}{3}\right)$ so, its equation is



$$y-\frac{1}{3}=-\frac{4}{11}\left(x+\frac{1}{3}\right)$$

$$\Rightarrow 12x+33y=7$$

18. (c) : We have, $s=\sqrt{1+t}$

$$\Rightarrow \ \frac{ds}{dt}=\frac{1}{2\sqrt{1+t}} \ \Rightarrow \ \frac{ds}{dt}=\frac{1}{2s}$$

$$\Rightarrow \ \frac{d^2s}{dt^2}=-\frac{1}{2s^2}\cdot\frac{ds}{dt}=-\frac{1}{2s^2}\times\frac{1}{2s}=-\frac{1}{4s^3}$$

$$\Rightarrow \ \frac{d^2s}{dt^2}=-2\times\left(\frac{1}{2s}\right)^3=-2\left(\frac{ds}{dt}\right)^3$$

19. (a) : Let $I=\int\sin(101x)\sin^{99}x\,dx$

$$\Rightarrow \ I=\int\sin(100x+x)\sin^{99}x\,dx$$

$$\Rightarrow \ I=\int\{\sin100x\cos x+\cos100x\sin x\}\sin^{99}x\,dx$$

$$\Rightarrow \ I=\int\sin100x\cos x\sin^{99}x\,dx +\int\cos100x\sin x\sin^{99}x\,dx$$

$$\Rightarrow \ I=\sin100x\int\sin^{99}x\cos x\,dx -\int\left\{\frac{d}{dx}(\sin100x)\int\sin^{99}x\cos x\,dx\right\}dx +\int\cos100x\sin x\sin^{99}x\,dx$$

$$\Rightarrow \ I=\sin100x\frac{\sin^{100}x}{100}-\int\cos100x\sin^{100}x\,dx +\int\cos100x\sin^{100}x\,dx$$

$$\Rightarrow \ I=\frac{1}{100}\sin(100x)(\sin x)^{100}+C$$

20. (a) : The equation of tangent at

$$P(x,y)\text{ is } Y-y=\frac{dy}{dx}(X-x)$$

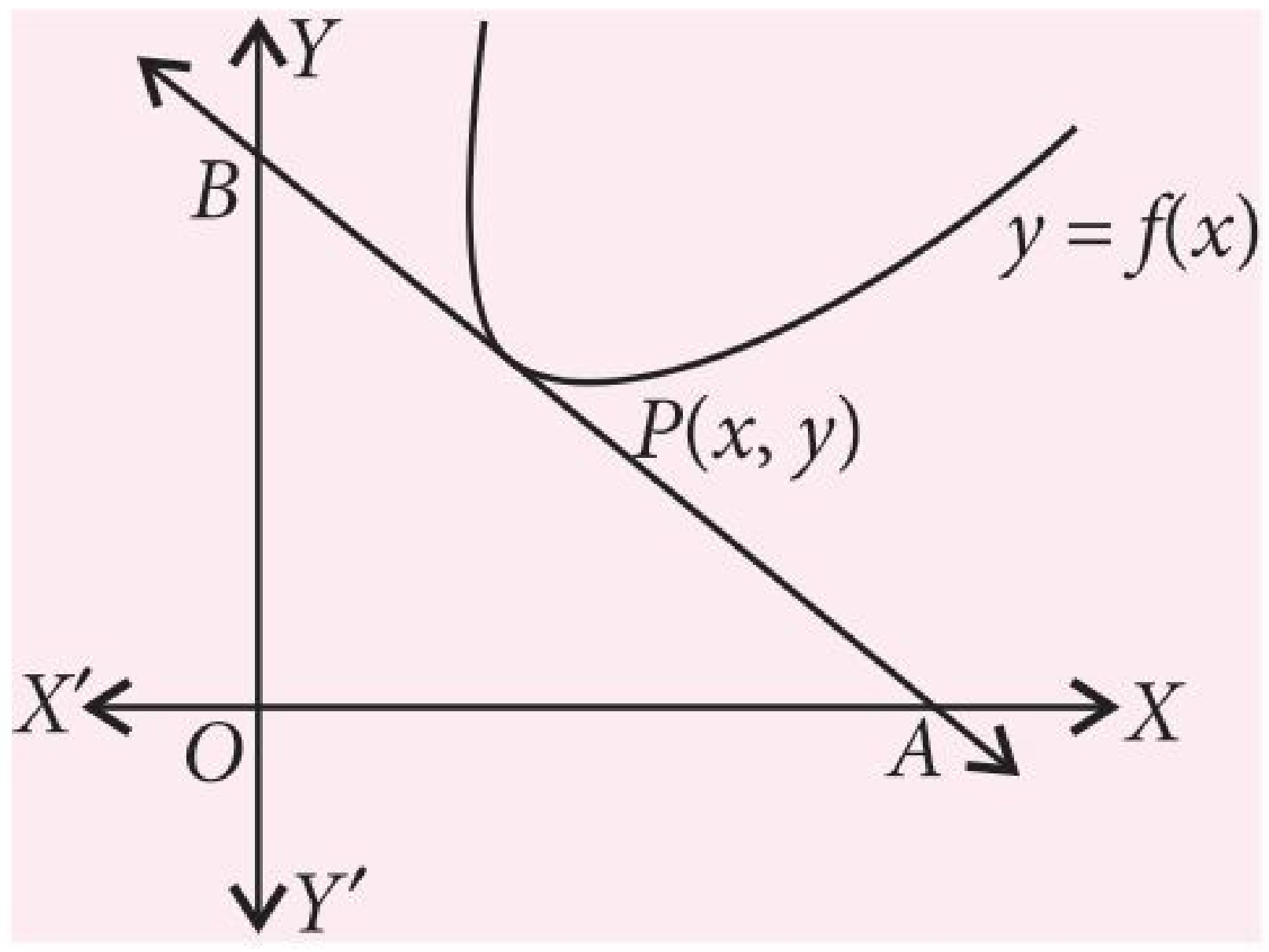
It cuts the coordinate axes at

$$A\left(x-y\frac{dx}{dy},0\right)\text{ and } B\left(0,y-x\frac{dy}{dx}\right)$$

It is given that $AP : BP = 3 : 1$

$$\therefore \ x=\frac{3\times0+x-y\frac{dx}{dy}}{3+1}\text{ and } y=\frac{3\left(y-x\frac{dy}{dx}\right)}{3+1}$$

$$\Rightarrow \ x-y\frac{dx}{dy}=4x\text{ and }3y-3x\frac{dy}{dx}=4y$$



$$\Rightarrow \ 3x\frac{dy}{dx}+y=0$$

$$\Rightarrow \ \frac{3}{y}dy+\frac{1}{x}dx=0$$

Integrating, we get

$y^3x = C$ ...(i)

It passes through (1, 1), $\therefore$ $C = 1$

Putting $C = 1$ in (i), we get $y^3x = 1$ as the equation of the curve. Clearly, it passes through (1/8, 2).

Also, equation of normal is given by

$$Y - y = -\frac{dx}{dy}(X-x).$$

$\therefore$ Equation of normal at (1, 1) is $3x - y - 2 = 0$.

21. (126) : Let x_1 be the number of stations before the first halting station, x_2 between first and second, x_3 between second and third, x_4 between third and fourth and x_5 on the right of the fourth station.

Then, $x_1 \geq 0$, $x_5 \geq 0$, $x_2, x_3, x_4 \geq 1$ such that

$$x_1 + x_2 + x_3 + x_4 + x_5 = 8 \qquad \text{...(i)}$$

The total number of ways is the number of solutions of the above equation.

Let $y_2 = x_2 - 1$, $y_3 = x_3 - 1$, $y_4 = x_4 - 1$. Then (i) becomes

$$x_1 + y_2 + y_3 + y_4 + x_5 = 5 \qquad \text{...(ii)}$$

where $y_2, y_3, y_4 \geq 0$

The number of solutions of (ii) are

$$^{5+5-1}C_{5-1} = {}^9C_4$$

22. (7) : We have $\left|\dfrac{z-6}{z-8}\right| = 1$

$\Rightarrow \quad |z - 6| = |z - 8|$

$\Rightarrow \quad z$ lies on the perpendicular bisector of the segment joining (6, 0) and (8, 0) *i.e.* on $x = 7$.

$\Rightarrow \quad z = 7 + iy$, where $y \in \mathbf{R}$ ...(i)

$$\text{Now, } z = x + i\begin{vmatrix} -3 & 2i & 2+i \\ -2i & 2 & 4-3i \\ 2-i & 4+3i & 7 \end{vmatrix} = x + 35i \qquad \text{...(ii)}$$

From (i) and (ii), we get $x = 7$ and $y = 35$.

23. (0) : Let $f(y) = y^2 - my + 1$. Since 1 lies between the roots of the equation $y^2 - my + 1 = 0$

$\therefore \quad m^2 - 4 > 0$ and $1 - m + 1 < 0$

[$\because$ Disc > 0 and $f(1) < 0$]

$\Rightarrow \quad (m < -2$ or $m > 2)$ and $m > 2 \Rightarrow m > 2$

Now, using A.M. – G.M. inequality, we get

$$\frac{|x|^2 + 16}{2} \geq 4|x| \;\Rightarrow\; 0 \leq \frac{4|x|}{|x|^2 + 16} \leq \frac{1}{2}$$

$$\Rightarrow\; 0 < \left(\frac{4|x|}{|x|^2 + 16}\right)^m < 1 \Rightarrow \left[\left(\frac{4|x|}{|x|^2 + 16}\right)^m\right] = 0.$$

24. (6): We find that cos40° + cos80° + cos160°

$= 2\cos 60°\cos 20° - \cos 20° = 0$ ...(i)

Now, cos40°cos80° + cos80°cos160° + cos160°cos40°

$$= \frac{1}{2}(2\cos 40°\cos 80° + 2\cos 80°\cos 160° + 2\cos 160°\cos 40°)$$

$$= \frac{1}{2}(\cos 120° + \cos 40° + \cos 240° + \cos 80° + \cos 200° + \cos 120°)$$

$$= \frac{1}{2}\left(-\frac{1}{2} + \cos 40° - \frac{1}{2} + \cos 80° + \cos 200° - \frac{1}{2}\right)$$

$$= \frac{1}{2}\left(-\frac{3}{2} + 2\cos 60°\cos 20° - \cos 20°\right) = -\frac{3}{4} \quad \text{[Using (i)]}$$

Also, cos40°cos80°cos160°

$$= \frac{\sin(2^3 \times 40°)}{2^3 \sin 40°} = \frac{\sin 320°}{8(\sin 40°)} = -\frac{1}{8}$$

$$\left[\because\; \cos\alpha\cos 2\alpha\cos 4\alpha \ldots \cos(2^{n-1}\alpha) = \frac{\sin 2^n\alpha}{2^n \sin\alpha} \text{ for all } n \in N\right]$$

So, the equation having cos40°, cos80° and cos160° as its roots is given by

$$x^3 - x^2 \times 0 + x \times \left(-\frac{3}{4}\right) - \left(-\frac{1}{8}\right) = 0 \;\text{ or }\; 8x^3 - 6x + 1 = 0$$

The equation having sec40°, sec80° and sec160° as its roots is $x^3 - 6x^2 + 8 = 0$

$\therefore \quad$ sec40° + sec80° + sec160° = 6

25. (1) : We have, $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$

and $\sin(\cos^{-1} x) = \sqrt{1 - x^2}$

$\therefore \quad \tan\{\sin^{-1}(\cos(\sin^{-1}x))\}\tan\{\cos^{-1}(\sin(\cos^{-1}x))\}$

$= \tan\{\sin^{-1}(\sqrt{1 - x^2})\}\tan\{\cos^{-1}(\sqrt{1 - x^2})\}$

SAMURAI SUDOKU

SOLUTION - JUNE 2022

6	5	1	4	9	8	3	2	7				4	2	3	1	5	7	9	8	6
2	4	3	6	7	1	5	9	8				8	1	9	6	4	3	2	7	5
8	9	7	2	5	3	6	1	4				7	5	6	8	9	2	3	1	4
4	6	9	1	2	7	8	3	5				6	4	1	7	3	8	5	9	2
7	3	2	8	4	5	9	6	1				9	7	2	5	1	6	8	4	3
1	8	5	9	3	6	4	7	2				5	3	8	4	2	9	1	6	7
5	2	4	3	1	9	7	8	6	1	5	2	3	9	4	2	7	1	6	5	8
3	7	8	5	6	2	1	4	9	3	8	7	2	6	5	9	8	4	7	3	1
9	1	6	7	8	4	2	5	3	9	6	4	1	8	7	3	6	5	4	2	9
						6	3	7	4	2	9	5	1	8						
						4	2	1	5	7	8	6	3	9						
						5	9	8	6	1	3	7	4	2						
1	3	8	2	5	6	9	7	4	2	3	1	8	5	6	2	1	7	4	9	3
6	2	7	9	3	4	8	1	5	7	9	6	4	2	3	9	6	8	1	5	7
9	5	4	7	1	8	3	6	2	8	4	5	9	7	1	5	3	4	2	6	8
5	6	3	4	7	2	1	8	9				6	1	9	8	2	5	7	3	4
8	4	2	1	9	3	7	5	6				5	8	4	3	7	6	9	2	1
7	1	9	8	6	5	2	4	3				7	3	2	1	4	9	5	8	6
4	7	6	3	2	1	5	9	8				1	6	5	7	8	2	3	4	9
3	9	5	6	8	7	4	2	1				2	4	7	6	9	3	8	1	5
2	8	1	5	4	9	6	3	7				3	9	8	4	5	1	6	7	2

$$= \tan\{\sin^{-1}(\sqrt{1-x^2})\}\tan\left\{\frac{\pi}{2}-\sin^{-1}(\sqrt{1-x^2})\right\}$$

$$= \tan\{\sin^{-1}(\sqrt{1-x^2})\}\cot\left\{\sin^{-1}(\sqrt{1-x^2})\right\}=1$$

26. (0.57) : Let V be the volume of the parallelepiped formed by the given vectors.

Let $\vec{\alpha}=\hat{i}+a\hat{j}+\hat{k},\ \vec{\beta}=\hat{j}+a\hat{k},\ \vec{\gamma}=a\hat{i}+\hat{k}$.

Then, $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=1+a^3-a$

$\therefore\ V=[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=1+a^3-a$

Now $\dfrac{dV}{da}=3a^2-1$ and $\dfrac{d^2V}{da^2}=6a$

For maxima or minima, we have $\dfrac{dV}{da}=0 \Rightarrow a=\pm\dfrac{1}{\sqrt{3}}$

and $\left(\dfrac{d^2V}{da^2}\right)_{a=1/\sqrt{3}}=\dfrac{6}{\sqrt{3}}>0$

So, V is minimum for $a=\dfrac{1}{\sqrt{3}}$.

27. (1) : We have, $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$

$\Rightarrow\ \vec{a}\times(\vec{a}\times\vec{b})=\vec{a}\times(\hat{j}-\hat{k})$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}=(\hat{i}+\hat{j}+\hat{k})\times(\hat{j}-\hat{k})$

$\Rightarrow\ (\hat{i}+\hat{j}+\hat{k})-3\vec{b}=-2\hat{i}+\hat{j}+\hat{k}\ \Rightarrow\ 3\vec{b}=3\hat{i}\ \Rightarrow\ \vec{b}=\hat{i}$

28. (5): We have, $x=4t^3+3$ and $y=4+3t^4$

$\therefore\ \dfrac{dx}{dt}=12t^2$ and $\dfrac{dy}{dt}=12t^3$

$\Rightarrow\ \dfrac{dy}{dx}=\dfrac{12t^3}{12t^2}=t\ \ \therefore\ \dfrac{dx}{dy}=\dfrac{1}{t}=t^{-1}$

$\Rightarrow\ \dfrac{d^2x}{dy^2}=-t^{-2}\dfrac{dt}{dy}=-t^{-2}\times\dfrac{1}{12t^3}=-\dfrac{1}{12}t^{-5}$

$\therefore\ \dfrac{d^2x}{dy^2}\Big/\left(\dfrac{dx}{dy}\right)^n=-\dfrac{1}{12}t^{-5}\times t^n=-\dfrac{1}{12}t^{n-5}$

Since $\dfrac{d^2x}{dy^2}\Big/\left(\dfrac{dx}{dy}\right)^n$ is constant.

Therefore, $n=5$.

29. (–3) : We know that the lines

$\dfrac{x-x_1}{l_1}=\dfrac{y-y_1}{m_1}=\dfrac{z-z_1}{n_1}$

and $\dfrac{x-x_2}{l_2}=\dfrac{y-y_2}{m_2}=\dfrac{z-z_2}{n_2}$ are coplanar, iff

$$\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=0$$

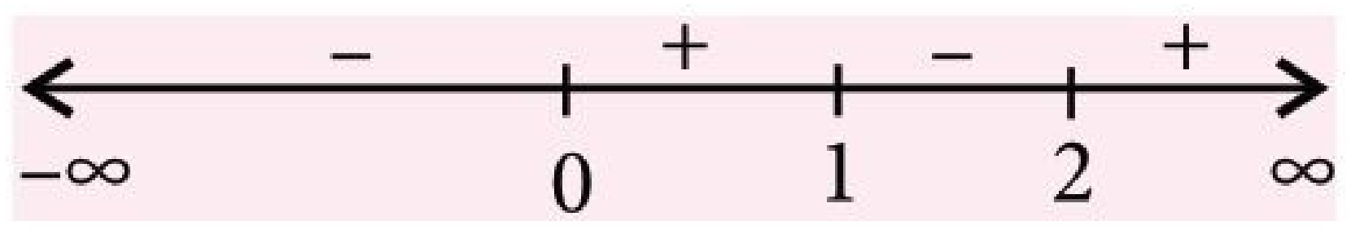
Since, the given lines are coplanar, then

$\begin{vmatrix}1-2 & 4-3 & 5-4\\ 1 & 1 & -k\\ k & 2 & 1\end{vmatrix}=0\ \Rightarrow\ k^2+3k=0\Rightarrow k=0,-3.$

30. (2) : We have, $f(x)=\displaystyle\int_1^x t(t^2-3t+2)dt$

$\Rightarrow\ f'(x)=x^3-3x^2+2x=x(x-2)(x-1)$

The change in signs of $f'(x)$ for different values of x are as follows :



Clearly $f(x)$ is decreasing in $[1, 2]$ and increasing in $[2, 3]$.

$\therefore\ \text{Min.} f(x)=f(2)=\displaystyle\int_1^2 t(t^2-3t+2)dt$

$=\left[\dfrac{t^4}{4}-t^3+t^2\right]_1^2=-\dfrac{1}{4}$

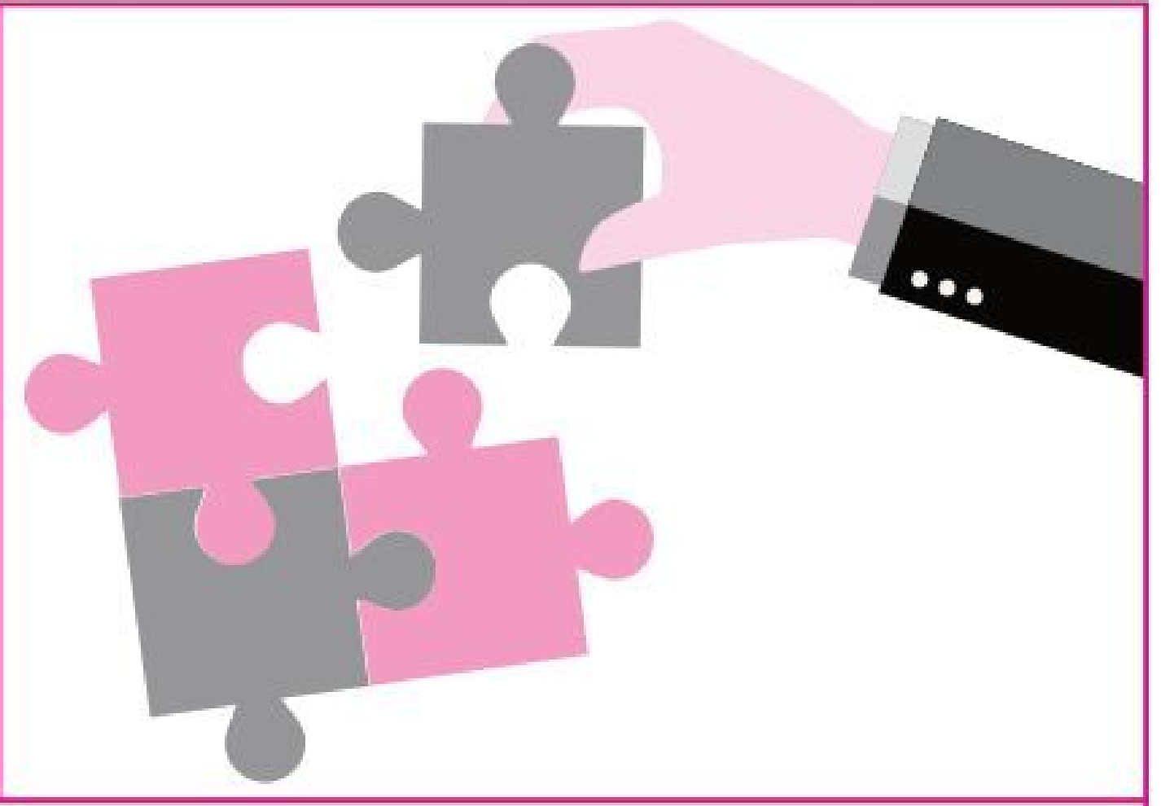
and $\max.\ f(x)=f(3)=\displaystyle\int_1^3 t(t^2-3t+2)dt$

$=\left[\dfrac{t^4}{4}-t^3+t^2\right]_1^3=2$

Since $f(x)$ is continuous on $[1, 3]$. Therefore, the range of $f(x)=[f(2),f(3)]=\left[-\dfrac{1}{4},2\right]$. $\therefore\ k=2.$

◆ ◆

PUZZLE CORNER



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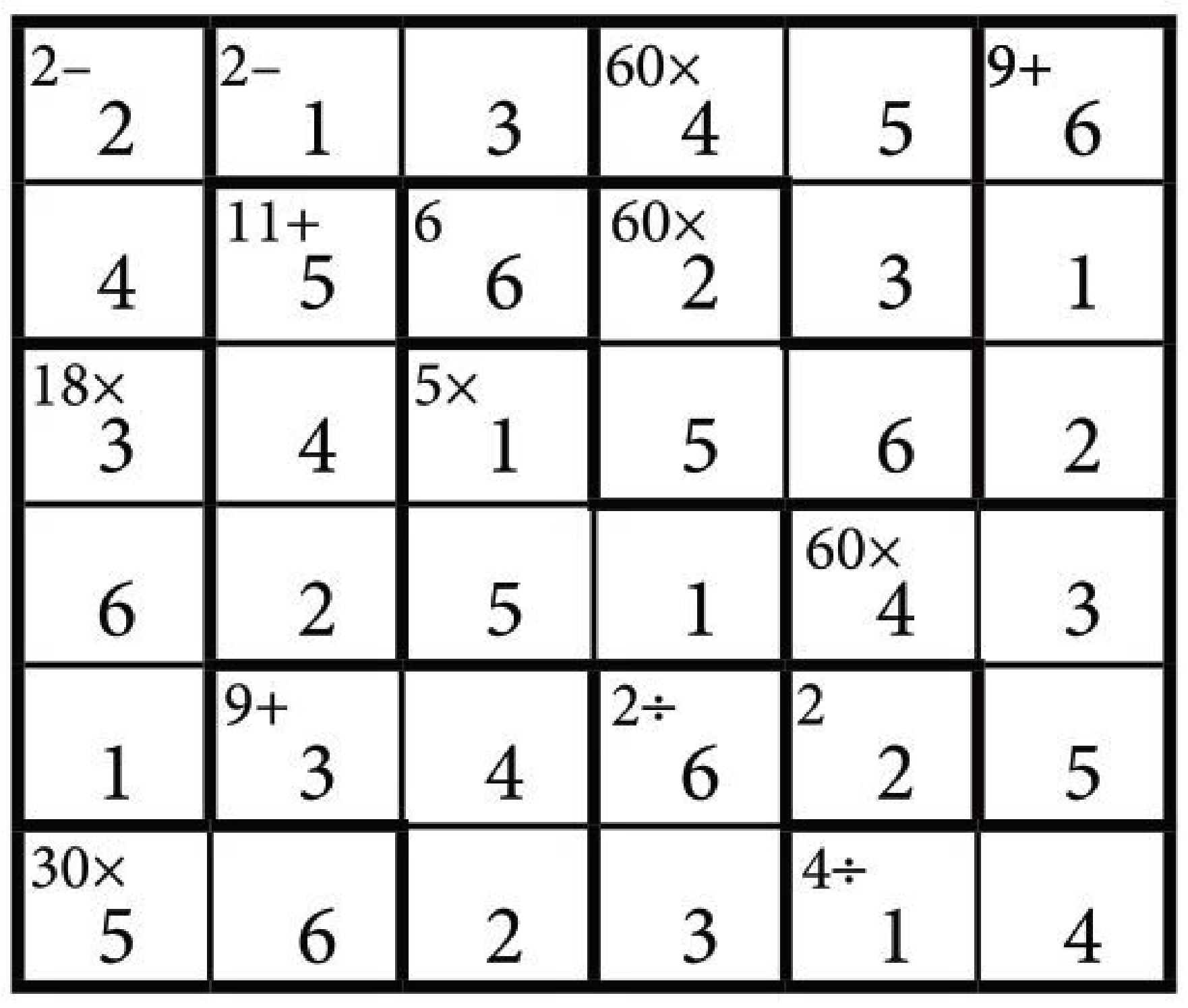


2– 2	2– 1	3	60× 4	5	9+ 6
4	11+ 5	6 6	60× 2	3	1
18× 3	4	5× 1	5	6	2
6	2	5	1	60× 4	3
1	9+ 3	4	2÷ 6	2 2	5
30× 5	6	2	3	4÷ 1	4

CUET (UG)

PRACTICE PAPER 2022

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Max. Marks : 200 Time : 45 minutes

Multiple Choice Questions (MCQs)

1. The function $f: R \rightarrow R$ defined by $f(x) = 6^x + 6^{|x|}$ is

(a) One-one and onto (b) Many-one and onto

(c) One-one and into (d) Many-one and into

2. The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1}x$ is

(a) 2 (b) $\frac{-1}{2\sqrt{1-x^2}}$

(c) $\frac{2}{x}$ (d) $1 - x^2$

3. Find the area above x-axis, bounded by the curves $y = 2^{kx}$, $x = 0$ and $x = 2$.

(a) $\frac{4^k - 1}{k\log_e 2}$ (b) $\frac{2^k - 1}{2\log_e 2}$

(c) $\frac{3-k}{k\log_e 2}$ (d) $\frac{-1+3^k}{2\log_e 2}$

4. $\tan^{-1}\left[\cos\left(2\tan^{-1}\frac{3}{4}\right)+\sin\left(2\cot^{-1}\frac{1}{2}\right)\right]$ is

(a) not defined (b) $\frac{\pi}{4}$

(c) $>\frac{\pi}{4}$ (d) $<\frac{\pi}{4}$

5. Evaluate : $\int\sqrt{(x-3)(5-x)}\,dx$

(a) $\frac{1}{2}(x-4)\sqrt{(x-3)(5-x)}+\frac{1}{2}\cos^{-1}(x-4)+C$

(b) $\frac{1}{2}(x-4)\sqrt{(x-3)(5-x)}+\frac{1}{2}\sin^{-1}(x-4)+C$

(c) $\frac{1}{2}\sqrt{(x-3)(5-x)}+\frac{1}{2}\sin^{-1}(x-4)+C$

(d) None of these

6. Given the differential equation

$\frac{dy}{dx}=\frac{6x^2}{2y+\cos y}$; $y(1)=\pi$.

Which of the following option is correct?

(a) Solution is $y^2 - \sin y = -2x^3 + C$

(b) Solution is $y^2 + \sin y = 2x^3 + C$

(c) $C = \pi^2 + 2\sqrt{2}$

(d) $C = \pi^2 + 2$

7. If $A=\begin{bmatrix}1 & 2 & x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & -2 & y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ and

$AB = I_3$, then $x + y$ equals

(a) 0 (b) –1

(c) 2 (d) None of these

8. If a, b, c are cube roots of unity,

then $\begin{vmatrix}e^a & e^{2a} & e^{3a}-1\\e^b & e^{2b} & e^{3b}-1\\e^c & e^{2c} & e^{3c}-1\end{vmatrix}=$

(a) 0 (b) e

(c) e^2 (d) e^3

9. Let $f(x) = |\sin x|$. Then

(a) f is everywhere differentiable

(b) f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$

(c) f is everywhere continuous but not differentiable at $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$

(d) None of these

10. Consider a binary operation $*$ on the set {1, 2, 3, 4, 5} given by the multiplication table.

Compute $(2 * 3) * (4 * 5)$.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

(a) 1 (b) 2 (c) 3 (d) 4

11. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}-3\hat{j}+6\hat{k}$.

(a) 5 (b) 6 (c) 7 (d) 8

12. The value of

$$\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right], x \in \left(0, \frac{\pi}{4}\right), \text{ is}$$

(a) $\pi - x$ (b) $\pi - \frac{x}{2}$

(c) $\pi - \frac{x}{3}$ (d) none of these

13. The vector equation of the plane passing through a point having position vector $2\hat{i}+3\hat{j}+4\hat{k}$ and perpendicular to the vector $2\hat{i}+\hat{j}-2\hat{k}$ is

(a) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=-1$

(b) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=8$

(c) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=9$

(d) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=15$

14. If $f(x) = x^3 - 6x^2 + 9x + 3$ be a decreasing function, then x lies in

(a) $(-\infty, -1) \cap (3, \infty)$ (b) $(1, 3)$

(c) $(3, \infty)$ (d) None of these

15. The solution of differential equation

$\frac{dy}{dx} - 3y = \sin 2x$ is

(a) $y = e^{-3x}\left[\frac{\cos 2x + 3\sin 2x}{13}\right]+c$

(b) $y = e^{-3x}\left(\frac{\cos 2x - 3\sin 2x}{13}\right)+c$

(c) $ye^{-3x} = -e^{-3x}\frac{(2\cos 2x + 3\sin 2x)}{13}+c$

(d) none of these

16. If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix}$, then find A^{-1}.

(a) $\begin{bmatrix} 1 & -4/3 & 1/2 \\ -1/3 & 7/6 & 5/6 \\ -1/2 & 5/3 & 1/6 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -4/3 & 1/3 \\ -1/2 & 7/6 & -1/6 \\ -1/2 & 5/6 & 1/6 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -4/3 & 1/2 \\ -1/2 & 7/6 & 2/3 \\ -1/2 & 5/3 & 1/4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -4/3 & 1/2 \\ -1/2 & 1 & 2/3 \\ -1/2 & 5/3 & 1/4 \end{bmatrix}$

17. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(a) 1 (b) 2 (c) 5 (d) $\frac{8}{3}$

18. Let $f(x)$ and $g(x)$ be differentiable functions for $0 \le x \le 1$, such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$. Let there exist a real number c in $(0, 1)$ such that $f'(c) = 2g'(c)$, then the value of $g(1)$ must be

(a) 1 (b) 2 (c) −2 (d) −1

19. The area bounded by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$ is

(a) 0 sq. unit

(b) $\left(\frac{8\sqrt{2}}{3}+\frac{9\pi}{2}-9\sin^{-1}\frac{1}{3}\right)$ sq. units

(c) 16π sq. units

(d) none of these

20. Let $f: R \to R$, $g: R \to R$ be two functions such that $f(x) = 2x - 3$, $g(x) = x^3 + 5$. The function $(fog)^{-1}(x)$ is equal to

(a) $\left(\frac{x+7}{2}\right)^{1/3}$ (b) $\left(x-\frac{7}{2}\right)^{1/3}$

(c) $\left(\frac{x-2}{7}\right)^{1/3}$ (d) $\left(\frac{x-7}{2}\right)^{1/3}$

21. Find the value of $\int_{-\pi/2}^{\pi/2} |\sin x|\, dx$.

(a) 0 (b) 1 (c) 2 (d) 3

22. The curve $y = 2e^{2x}$ intersects the y-axis at an angle $\cot^{-1}\left|\frac{8n-4}{3}\right|$, then sum of possible values of 'n' will be equal to

(a) 1 (b) −4 (c) 4 (d) 2

23. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, then find $(\text{adj } A)^{-1}$.

(a) A (b) $-A$
(c) A' (d) None of these

24. A manufacturer makes two products, A and B. Product A sells at ₹ 200 per unit and takes 30 minutes to make. Product B sells at ₹ 300 per unit and takes 1 hour to make. There is a permanent order of 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and the weekly turnover must not be less than ₹ 10000. If the profit on each of product A is ₹ 20 and on product B is ₹ 30, then how many product A and B respectively should be produced so that the profit is maximum? Also, find the maximum profit.

(a) 14, 33, ₹ 1270 (b) 26, 16, ₹ 1500
(c) 48, 16, ₹ 1440 (d) 14, 24, ₹ 1000

25. If $u = x^2 + y^2$ and $x = s + 3t$, $y = 2s - t$, then $\frac{d^2u}{ds^2}$ is equal to

(a) 12 (b) 32 (c) 36 (d) 10

26. The equation of the plane through the point (2, 5, – 3) perpendicular to the planes $x + 2y + 2z = 1$ and $x - 2y + 3z = 4$ is

(a) $3x - 4y + 2z - 20 = 0$
(b) $7x - y + 5z = 30$
(c) $x - 2y + z = 11$
(d) $10x - y - 4z = 27$

27. $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] =$

(a) $\sqrt{\frac{x^2+1}{x^2+2}}$ (b) $\sqrt{\frac{x^2-1}{x^2-2}}$
(c) $\sqrt{\frac{x-1}{x-2}}$ (d) $\sqrt{\frac{x+1}{x+2}}$

28. The bag 'A' contains 5 white and 3 black balls while the bag 'B' contains 4 white and 7 black balls. One of the bags is chosen at random and a ball is drawn from it. What is the probability that the ball is white?

(a) $\frac{87}{176}$ (b) $\frac{81}{172}$ (c) $\frac{82}{179}$ (d) $\frac{87}{179}$

29. Let R be the relation on the set of all real numbers defined by aRb iff $|a - b| \leq 1$. Then, R is

(a) Reflexive and symmetric
(b) Symmetric only
(c) Transitive only
(d) Anti-symmetric only

30. The area given by $x + y \leq 6$, $x^2 + y^2 \leq 6y$ and $y^2 \leq 8x$ is

(a) $\left(\frac{27\pi+2}{12}\right)$ sq. units (b) $\left(\frac{27\pi-2}{12}\right)$ sq. units
(c) $\left(\frac{27\pi-1}{6}\right)$ sq. units (d) $\left(\frac{27\pi+1}{6}\right)$ sq. units

31. Evaluate : $\int \frac{x^3}{x+2}dx$

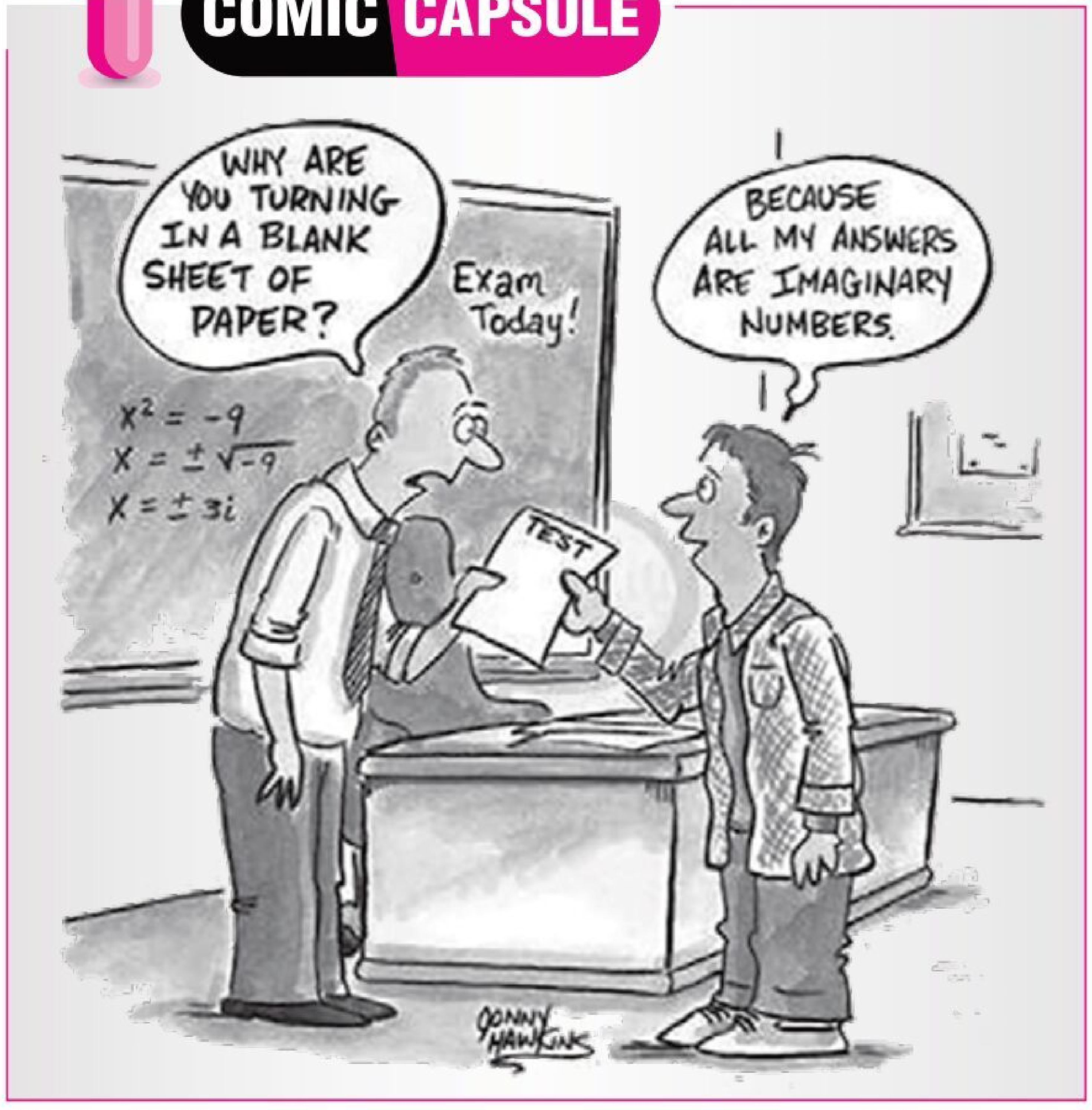
(a) $\frac{x^3}{3} - x^2 - 4x - 8\log|x+2| + C$

(b) $\frac{x^3}{3} - x^2 + 4x - 8\log|x+2| + C$

(c) $\frac{x^3}{3} + x^2 + 4x + 8\log|x+2| + C$

(d) $\frac{x^3}{3} + x^2 + 4x - 8\log|x+2| + C$

COMIC CAPSULE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Exam Today!
x² = -9
x = ± √-9
x = ± 3i
TEST
BECAUSE ALL MY ANSWERS ARE IMAGINARY NUMBERS.
JONNY HAWKINS

32. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and

$10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then α is

(a) -2 (b) 1

(c) 2 (d) 5

33. If $f(x) = \begin{vmatrix} (1+x)^{17} & (1+x)^{19} & (1+x)^{23} \\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34} \\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47} \end{vmatrix} = A$

$+ Bx + Cx_2 + \ldots$, then

(a) $A = 0$ (b) $B = 1$

(c) $C = 0$ (d) $D = 1$

34. A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate if the radius of the plate before heating is 10 cm.

(a) 3.03π (b) 5.43π (c) 2π (d) 4π

Assertion & Reasoning Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.

(c) If Statement-I is true but Statement-II is false.

(d) If Statement-I is false and Statement-II is true.

35. Let $I = \int \frac{\sqrt{\cos 2x}}{\sin x} dx$

Statement-I : $I = \ln\left[\left(\frac{1-\sqrt{1-t^2}}{t}\right)\left(\frac{\sqrt{2}+\sqrt{1-t^2}}{\sqrt{2}-\sqrt{1-t^2}}\right)^{\frac{1}{\sqrt{2}}}\right] + C,$

where $t = \tan x$

Statement-II : $\int \cot\theta \, d\theta = \log|\sin\theta| + C$

36. Statement-I : Let $A = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$, $B = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$, then $(A + B)(A - B)$ equals $A^2 - B^2$.

Statement-II : $AB = BA$.

37. Statement-I : The unit vector in the direction of sum of the vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}-\hat{k}$ and $2\hat{j}+6\hat{k}$ is $-\frac{1}{7}(3\hat{i}+2\hat{j}+6\hat{k})$.

Statement-II : Let $\vec{a}$ be a non-zero vector, then $\frac{\vec{a}}{|\vec{a}|}$ is a unit vector parallel to $\vec{a}$.

38. Statement-I : The area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$ is $\frac{7}{2}$ sq. units.

Statement-II : The area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 < 9\}$ is $\frac{9\pi}{8} + \frac{1}{3\sqrt{2}} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$.

Match the Following

39. Match the following :

Column I		**Column II**	
(P)	The value of $\begin{vmatrix} x+2 & x+3 & x+5 \\ x+4 & x+6 & x+9 \\ x+8 & x+11 & x+15 \end{vmatrix}$ is	(1)	1
(Q)	If one of the roots of the equation $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix} = 0$ is – 2, then sum of all other five roots is	(2)	–6
(R)	The value of $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}\,i & 3\sqrt{2}+\sqrt{6}\,i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}\,i & \sqrt{27}+2i \end{vmatrix}$ is	(3)	2
(S)	If $f(\theta) =$ $\begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$ then $f(\pi/3) =$	(4)	–2

	P	Q	R	S		P	Q	R	S
(a)	4	3	2	1	(b)	1	2	3	4
(c)	2	1	4	3	(d)	3	4	1	2

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40. Match the following :

Column I		Column II	
(P)	$\frac{x-2}{3}=\frac{y-7}{4}=\frac{z+5}{2}$	(1)	perpendicular to the plane $3x+4y+2z=1$
(Q)	$\frac{x-1}{2}=\frac{y+1}{5}=\frac{z+1}{1}$	(2)	lies in the plane $7x-y-z=35$
(R)	$\frac{x-5}{1}=\frac{y+2}{3}=\frac{z-2}{4}$	(3)	has directions cosines $\frac{2}{\sqrt{30}},\frac{5}{\sqrt{30}},\frac{1}{\sqrt{30}}$
(S)	$\frac{x+1}{3}=\frac{y-2}{4}=\frac{z-7}{2}$	(4)	passes through $(5, 11, -3)$

	P	Q	R	S		P	Q	R	S
(a)	1	2	3	4	(b)	3	2	4	1
(c)	4	2	1	3	(d)	4	3	2	1

Case Based MCQs

Case I : Read the following and answer any four questions from 41 to 45 given below.

A magazine company in a town has 5000 subscribers on its list and collects fix charges of ₹ 3000 per year from each subscriber. The company proposes to increase the annual charges and it is believed that for every increase of ₹ 1, one subscriber will discontinue service.

41. If x denote the amount of increase in annual charges, then revenue R, as a function of x can be represented as

(a) $R(x) = 3000 \times 5000 \times x$

(b) $R(x) = (3000 - 2x)(5000 + 2x)$

(c) $R(x) = (5000 + x)(3000 - x)$

(d) $R(x) = (3000 + x)(5000 - x)$

42. If magazine company increases ₹ 500 as annual charges, then R is equal to

(a) ₹ 15750000 (b) ₹ 16750000

(c) ₹ 17500000 (d) ₹ 15000000

43. If revenue collected by the magazine company is ₹ 15640000, then value of amount increased as annual charges for each subscriber, is

(a) 400

(b) 1600

(c) Both (a) and (b)

(d) None of these

44. What amount of increase in annual charges will bring maximum revenue?

(a) ₹ 1000 (b) ₹ 2000

(c) ₹ 3000 (d) ₹ 4000

45. Maximum revenue is equal to

(a) ₹ 15000000 (b) ₹ 16000000

(c) ₹ 20500000 (d) ₹ 25000000

Case II : Read the following passage and answer the questions from 46 to 50.

In a college hostel accommodating 1000 students, one of the hostellers came in carrying Corona virus, and the hostel was isolated. The rate at which the virus spreads is assumed to be proportional to the product of the number of infected students and remaining students. There are 50 infected students after 4 days.

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (c) **2.** (b) **3.** (a) **4.** (a) **5.** (c)
6. (d) **7.** (b, d) **8.** (a, b) **9.** (a, b, c, d)
10. (b, d) **11.** (a, b, c) **12.** (a, d) **13.** (a, b, c, d)
14. (a) **15.** (b) **16.** (a) **17.** (3) **18.** (5)
19. (5) **20.** (1)

46. If $n(t)$ denote the number of students infected by Corona virus at any time t, then maximum value of $n(t)$ is

(a) 50 (b) 100 (c) 500 (d) 1000

47. $\frac{dn}{dt}$ is proportional to

(a) $n(1000 - n)$ (b) $n(1000 + n)$
(c) $n(100 - n)$ (d) $n(100 + n)$

48. The value of $n(4)$ is

(a) 1 (b) 50
(c) 100 (d) 1000

49. The most general solution of differential equation formed in given situation is

(a) $\frac{1}{1000}\log\left(\frac{1000-n}{n}\right)=\lambda t+c$

(b) $\log\left(\frac{n}{100-n}\right)=\lambda t+c$

(c) $\frac{1}{1000}\log\left(\frac{n}{1000-n}\right)=\lambda t+c$

(d) None of these

50. The value of n at any time is given by

(a) $n(t)=\frac{1000}{1+999e^{-0.9906t}}$

(b) $n(t)=\frac{1000}{1-999e^{-0.9906t}}$

(c) $n(t)=\frac{100}{1-999e^{-0.996t}}$

(d) $n(t)=\frac{100}{999+e^{1000t}}$

ANSWER KEYS

1.	(d)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(b)
6.	(b)	**7.**	(a)	**8.**	(a)	**9.**	(b)	**10.**	(a)
11.	(a)	**12.**	(b)	**13.**	(a)	**14.**	(b)	**15.**	(c)
16.	(b)	**17.**	(b)	**18.**	(b)	**19.**	(b)	**20.**	(d)
21.	(c)	**22.**	(a)	**23.**	(a)	**24.**	(c)	**25.**	(d)
26.	(d)	**27.**	(a)	**28.**	(a)	**29.**	(a)	**30.**	(b)
31.	(b)	**32.**	(d)	**33.**	(a)	**34.**	(d)	**35.**	(d)
36.	(a)	**37.**	(d)	**38.**	(b)	**39.**	(a)	**40.**	(d)
41.	(d)	**42.**	(a)	**43.**	(c)	**44.**	(a)	**45.**	(b)
46.	(d)	**47.**	(a)	**48.**	(b)	**49.**	(c)	**50.**	(a)

⁂

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JEE WORK OUTS

1. Let (x_0, y_0) be the solution of the following equations $(2x)^{\ln 2} = (3y)^{\ln 3}$ and $3^{\ln x} = 2^{\ln y}$. Then x_0 is

(a) 1/6 (b) 1/3

(c) 1/2 (d) 6

2. Let $S(K) = 1 + 3 + 5 + ... + (2K - 1) = 3 + K^2$. Then which of the following is true?

(a) $S(1)$ is correct

(b) Principle of mathematical induction can be used to prove the formula

(c) $S(K) \nRightarrow S(K+1)$

(d) $S(K) \Rightarrow S(K+1)$

3. The real part of $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is

(a) $\frac{1}{3+5\cos\theta}$ (b) $\frac{1}{5-3\cos\theta}$

(c) $\frac{1}{3-5\cos\theta}$ (d) $\frac{1}{5+3\cos\theta}$

4. a, g, h are arithmetic mean, geometric mean and harmonic mean between two positive numbers x and y respectively. Then identify the correct statement among the following

(a) h is the harmonic mean between a and g

(b) No such relation exists between a, g and h

(c) g is the geometric mean between a and h

(d) a is the arithmetic mean between g and h

5. If $s_r = \alpha^r + \beta^r + \gamma^r$, then the value of $\begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix}$ is equal to

(a) 0 (b) $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

(c) $(\alpha + \beta + \gamma)^6$ (d) $(\alpha - \beta)^2(\beta - \gamma)^2(\gamma - \alpha)^2$

6. p : Sita is beautiful; q = Sita is clever.
When $\sim p \vee q$ is written in verbal form it becomes

(a) Sita is not beautiful or Sita is not clever

(b) Sita is beautiful and she is clever

(c) Sita is not beautiful or Sita is clever

(d) Sita is beautiful and Sita is not clever

7. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is

(a) $\frac{10!}{6}$ (b) 3! 7!

(c) ${}^{10}P_3 \cdot 7!$ (d) none of these

8. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$, then ab =

(a) 1 (b) 1/2 (c) 2 (d) 3

9. If the probability density function of a random variable X is $f(x) = x/2$ in $0 \le x \le 2$, then $P(X > 1.5 \mid X > 1)$ is equal to

(a) 7/16 (b) 3/4 (c) 7/12 (d) 21/64

10. The product of perpendiculars drawn from the origin to the lines represented by the equation, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, will be

(a) $\frac{ab}{\sqrt{a^2 - b^2 + 4h^2}}$ (b) $\frac{bc}{\sqrt{a^2 - b^2 + 4h^2}}$

(c) $\frac{ca}{\sqrt{(a^2 + b^2) + 4h^2}}$ (d) $\frac{c}{\sqrt{(a-b)^2 + 4h^2}}$

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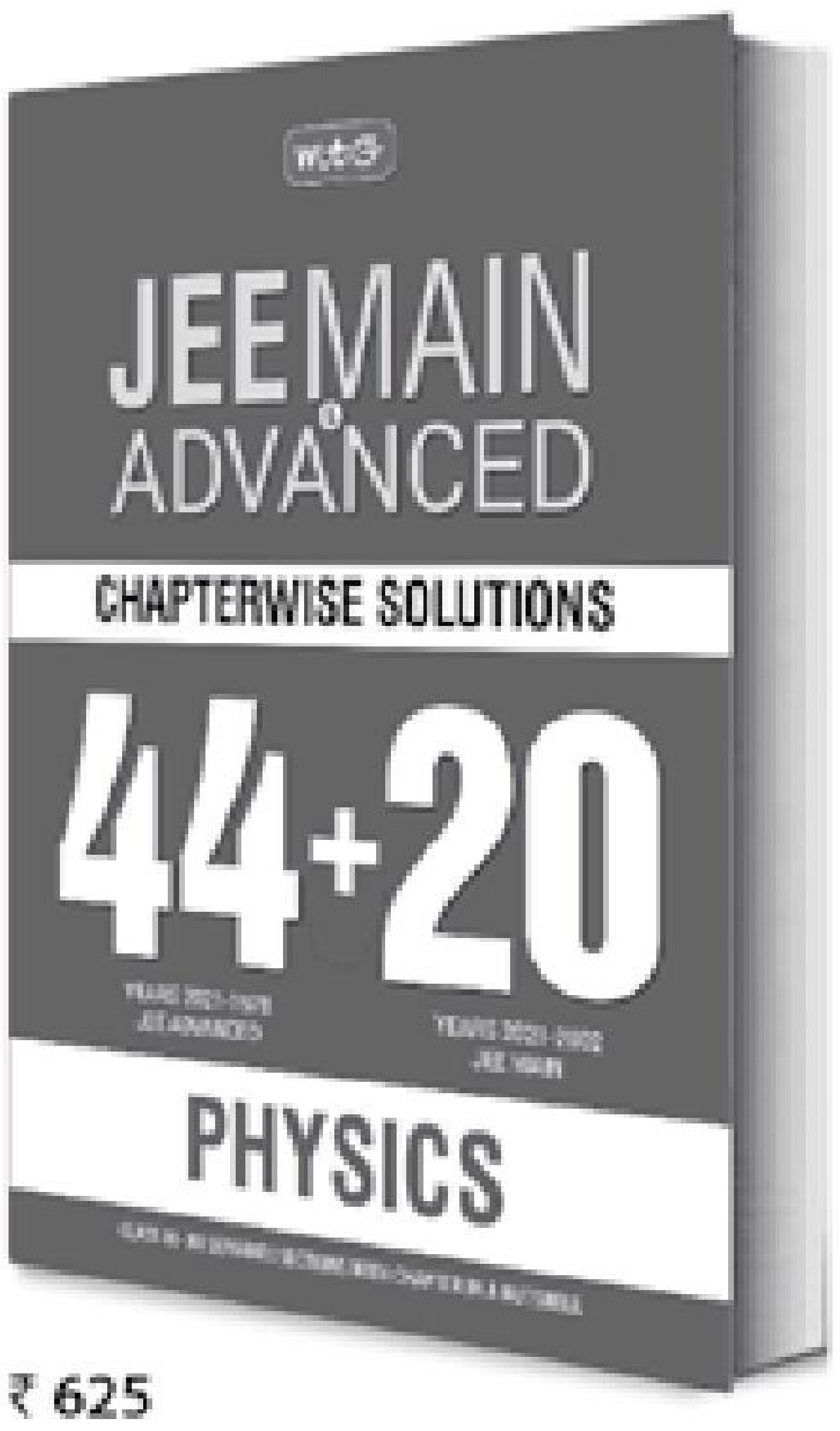
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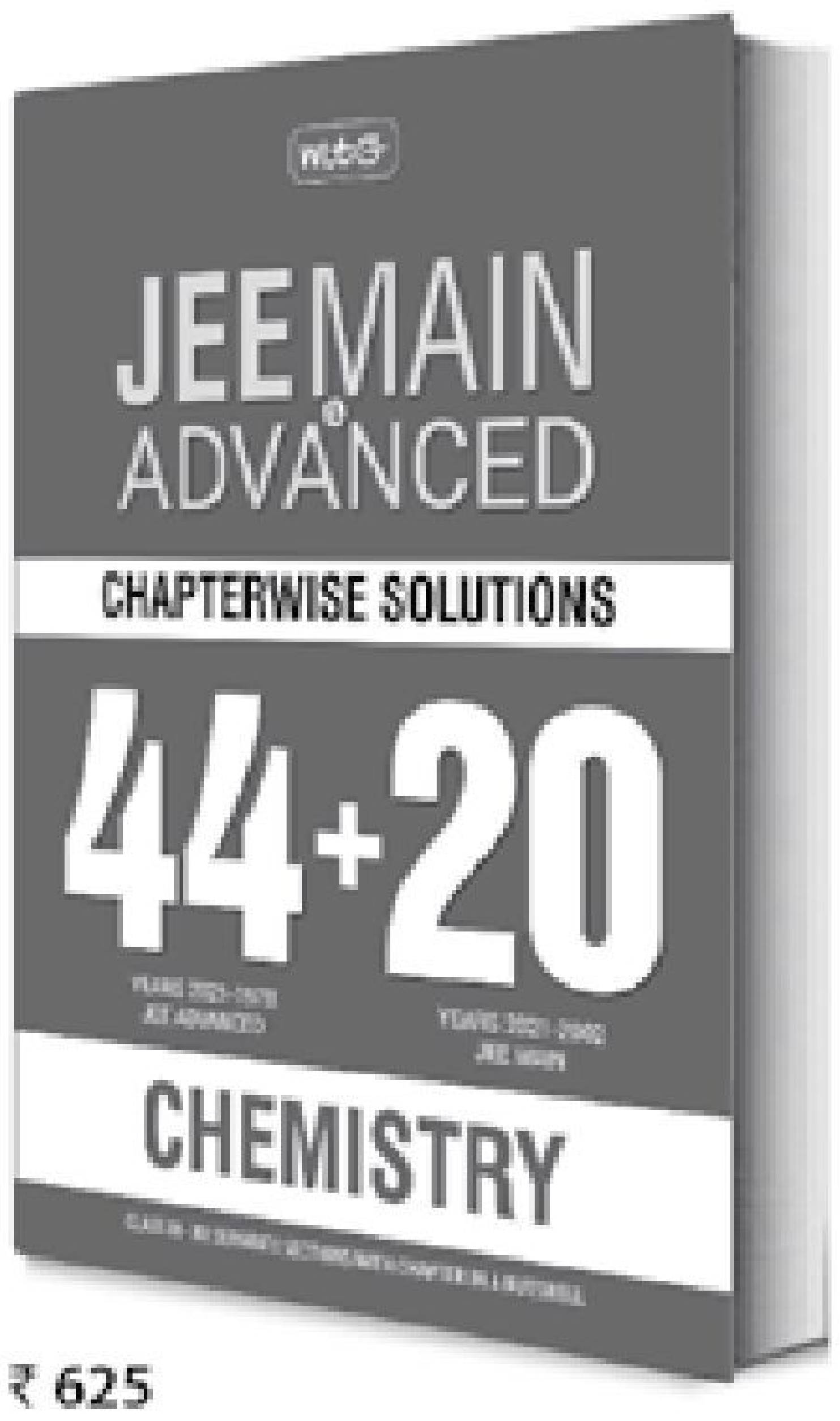
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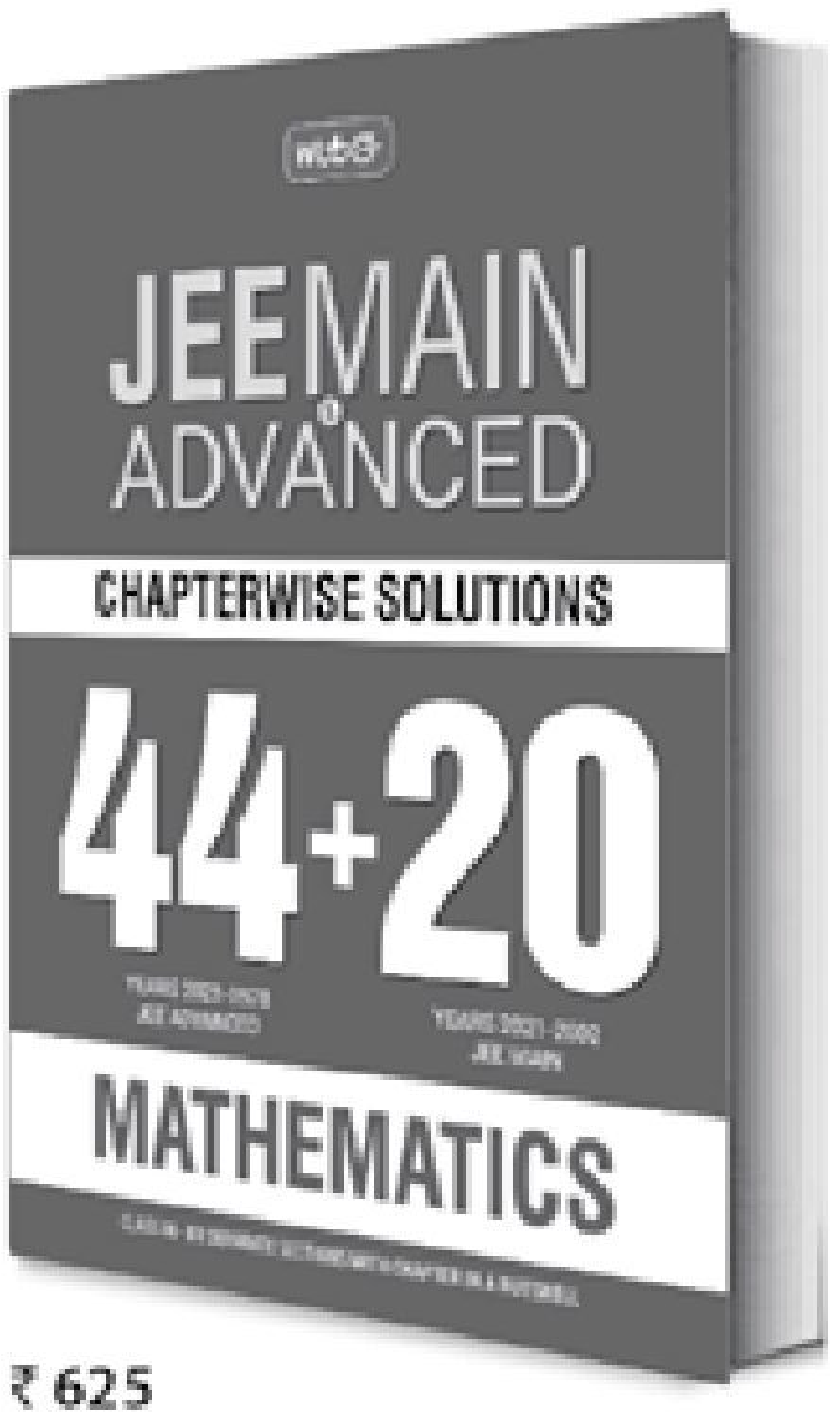
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11. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, (where $pq \neq 0$) are bisected by the x-axis, then

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$
(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

12. A circle C_1 of radius 2 touches both x-axis and y-axis. Another circle C_2 whose radius is greater than 2 touches circle C_1 and both the axes. Then the radius of circle C_2 is

(a) $6-4\sqrt{2}$ (b) $6+4\sqrt{2}$
(c) $6-4\sqrt{3}$ (d) $6+4\sqrt{3}$

13. If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then $\sin\alpha + \cos\alpha$ and $\sin\alpha - \cos\alpha$ must be equal to

(a) $\sqrt{2}\cos\theta, \sqrt{2}\sin\theta$ (b) $\sqrt{2}\sin\theta, \sqrt{2}\cos\theta$
(c) $\sqrt{2}\sin\theta, \sqrt{2}\sin\theta$ (d) $\sqrt{2}\cos\theta, \sqrt{2}\cos\theta$

14. A pair of straight lines drawn through the origin form with the line $2x + 3y = 6$ an isosceles right angled triangle, then the area of the triangle thus formed is

(a) $\Delta = \dfrac{36}{13}$ (b) $\Delta = \dfrac{12}{17}$
(c) $\Delta = \dfrac{13}{5}$ (d) none of these

15. Point D, E are taken on the side BC of a triangle ABC such that $BD = DE = EC$. If $\angle BAD = x$, $\angle DAE = y$ and $\angle EAC = z$, then the value of $\dfrac{\sin(x+y)\sin(y+z)}{\sin x \sin z} =$

(a) 1 (b) 2
(c) 4 (d) none of these

16. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $x + y + z$ is equal to

(a) xyz (b) 0 (c) 1 (d) $2xyz$

17. From the bottom of a pole of height h, the angle of elevation of top of a tower is α and the pole subtends angle β at the top of the tower. The height of the tower is

(a) $\dfrac{h\tan(\alpha-\beta)}{\tan(\alpha-\beta)-\tan\alpha}$ (b) $\dfrac{h\cot(\alpha-\beta)}{\cot(\alpha-\beta)-\cot\alpha}$
(c) $\dfrac{\cot(\alpha-\beta)}{\cot(\alpha-\beta)-\cot\alpha}$ (d) none of these

18. If f is an even function defined on the interval $(-5, 5)$, then four real values of x satisfying the equation $f(x) = f\left(\dfrac{x+1}{x+2}\right)$ are

(a) $\dfrac{-3-\sqrt{5}}{2}, \dfrac{-3+\sqrt{5}}{2}, \dfrac{3-\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}$

(b) $\dfrac{-5+\sqrt{3}}{2}, \dfrac{-3+\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}, \dfrac{3-\sqrt{5}}{2}$

(c) $\dfrac{3-\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}, \dfrac{-3-\sqrt{5}}{2}, \dfrac{5+\sqrt{3}}{2}$

(d) $-3-\sqrt{5}, -3+\sqrt{5}, 3-\sqrt{5}, 3+\sqrt{5}$

19. The value of $\lim\limits_{x\to\pi/2} \dfrac{\int_{\pi/2}^{x} t\,dt}{\sin(2x-\pi)}$ is

(a) ∞ (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/8$

20. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$ then $f^2(4) + g^2(4)$ is

(a) 8 (b) 16 (c) 32 (d) 64

21. The value of p for which the function

$$f(x) = \begin{cases} \dfrac{(4^x-1)^3}{\sin\dfrac{x}{p}\log_e\left(1+\dfrac{x^2}{3}\right)}, & x \neq 0 \\ 12(\log_e 4)^3, & x = 0 \end{cases}$$

is continuous at $x = 0$, is

(a) 1 (b) 2
(c) 3 (d) none of these

22. Maximize $z = 3x + y$ subject to $x + y \leq 6$, $y \leq 4$, $x \geq 1$, $x \geq 0$ and $y \geq 0$.

(a) 20 (b) 18 (c) 10 (d) 15

23. Let $f(x) = \int_0^x \dfrac{\cos t}{t}dt, x > 0$ then $f(x)$ has

(a) maxima when $n = -2, -4, -6, ...$
(b) maxima when $n = -1, -3, -5, ...$
(c) maxima when $n = 1, 3, 5, ...$
(d) none of these

24. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral $PQRS$ must be a

(a) parallelogram, which is neither a rhombus nor a rectangle
(b) square
(c) rectangle, but not a square
(d) rhombus, but not a square

25. Let $A = \{2, 3, 4, 5\}$ and let $R = \{(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)\}$ be a relation in A. Then R is

(a) reflexive and transitive
(b) reflexive and symmetric
(c) transitive and symmetric
(d) equivalence

26. The value of a so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+a\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is

(a) -3 (b) 3 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

27. $\int \frac{dx}{(1+x^2)\sqrt{p^2+q^2(\tan^{-1}x)^2}} =$

(a) $\frac{1}{q}\log\left|q\tan^{-1}x+\sqrt{p^2+q^2(\tan^{-1}x)^2}\right|+c$

(b) $\log\left|q\tan^{-1}x+\sqrt{p^2+q^2(\tan^{-1}x)^2}\right|+c$

(c) $\frac{2}{3q}(p^2+q^2\tan^{-1}x)^{3/2}+c$

(d) none of these

28. Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is

(a) 80 (b) 50 (c) $82\frac{1}{2}$ (d) 48

29. A tangent to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the coordinate axes at P and Q. The least value of PQ is

(a) $a - b$ (b) $a + b$ (c) $\frac{a+b}{2}$ (d) $2a$

30. The plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, 1, 1)$ and $(1, -1, 2)$ makes intercepts on the co-ordinate axes, the sum whose length is

(a) 3 (b) 4 (c) 6 (d) 12

ANSWERS

1.	(c)	**2.**	(d)	**3.**	(d)	**4.**	(c)	**5.**	(d)
6.	(c)	**7.**	(a)	**8.**	(a)	**9.**	(c)	**10.**	(d)
11.	(d)	**12.**	(b)	**13.**	(a)	**14.**	(a)	**15.**	(c)
16.	(a)	**17.**	(b)	**18.**	(a)	**19.**	(c)	**20.**	(c)
21.	(d)	**22.**	(b)	**23.**	(b)	**24.**	(a)	**25.**	(b)
26.	(c)	**27.**	(a)	**28.**	(b)	**29.**	(b)	**30.**	(d)

◆◆

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CONCEPT
MAP

BINOMIAL THEOREM

Class XI

Binomial Theorem for Fractional Index

- $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots + \frac{n(n-1)\ldots(n-r+1)x^r}{r!} + \ldots + \infty, |x| < 1$
- $T_{r+1} = \frac{n(n-1)(n-2)\ldots.(n-r+1)}{r!} x^r$
- T_{r+1} in $(1+x)^{-n} = \frac{(-1)^r n(n+1)(n+2)\ldots(n+r-1)x^r}{r!}$
- T_{r+1} in $(1-x)^{-n} = \frac{n(n+1)(n+2)\ldots.(n+r-1)x^r}{r!}$

Multinomial Theorem

$$(x_1 + x_2 + \ldots. + x_r)^n = \sum \frac{n!}{p_1! p_2! \ldots p_r!} x_1^{p_1} x_2^{p_2} \ldots x_r^{p_r},$$

where $p_1 + p_2 + p_3 + \ldots + p_r = n$

Binomial Coefficients

Greatest Binomial Coefficient in $(1 + x)^n$ are given by

- ${}^nC_{\frac{n}{2}}$, if n is even
- ${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$, if n is odd

Coefficient of

- $(r + 1)^{th}$ term in $(1 + x)^n$ is ${}^nC_r$
- x^r in $(1 + x)^n$ is ${}^nC_r$
- x^r in $(1 - x)^n$ is $(-1)^r\,{}^nC_r$
- $(r + 1)^{th}$ term in $(1 - x)^n$ is $(-1)^r\,{}^nC_r$

Number of Terms in the Expansion

- Number of terms in the expansion of $(x + y + z)^n$ where n is a positive integer.
  Now $(x + y + z)^n = x^n + {}^nC_1 x^{n-1}(y + z) + {}^nC_2 x^{n-2}(y + z)^2 + \ldots + {}^nC_n (y + z)^n \quad \ldots(i)$
  Now the number of terms in the $1^{st}, 2^{nd}, 3^{rd} \ldots (n + 1)^{th}$ terms on the R.H.S. of (i) are $1, 2, 3, \ldots n + 1$ respectively.
  Hence, total number of terms are
  $$1 + 2 + 3 + \ldots + (n + 1) = \sum_{r=1}^{n+1} r = \frac{1}{2}(n+1)(n+2)$$
- The number of terms in $(x_1 + x_2 + x_3 + \ldots..+ x_k)^n$ is equal to the number of non-negative integral solutions of the equation $r_1 + r_2 + \ldots. + r_k = n$
  The number of such solutions $= {}^{n+k-1}C_{k-1}$.

Some Important Results

- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1} = \frac{n}{r}\left(\frac{n-1}{r-1}\right){}^{n-2}C_{r-2}$
- ${}^nC_x = {}^nC_y \Rightarrow$ Either $x = y$ or $x + y = n$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- $\sum_{r=0}^{n} {}^nC_r = \sum_{r=0}^{n} C_r = 2^n$
- $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$
- $\sum_{r=0}^{n} r \cdot C_r = n2^{n-1}$
- $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots. + (-1)^n C_n^2 = \begin{cases} 0, n \text{ is odd} \\ (-1)^{\frac{n}{2}}\,{}^nC_{\frac{n}{2}}, \ n \text{ is even} \end{cases}$
- $C_0 - C_2 + C_4 - C_6 + \ldots = 2^{\frac{n}{2}} \cos\frac{n\pi}{4}$
- $C_1 - C_3 + C_5 - C_7 + \ldots = 2^{\frac{n}{2}} \sin\frac{n\pi}{4}$

Middle Terms

- No. of terms $= n + 1$
- If n is even, then $T_{\frac{n}{2}+1}$ is the middle term.
- If n is odd, then $T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$ are two middle terms.

General Term

- $T_{r+1} = {}^nC_r x^{n-r} a^r$
- T_{r+1} from the end
  $= T_{n-r+1}$ from the beginning

MATRICES

Class XII

CONCEPT
MAP

Operations		Properties
Addition and Subtraction	$A \pm B = C$ i.e., $[a_{ij}]_{m \times n} \pm [b_{ij}]_{m \times n} = [a_{ij} \pm b_{ij}]_{m \times n} = [c_{ij}]_{m \times n}$	• $A + B = B + A$ • $A + (B + C) = (A + B) + C$ • Additive inverse = $-A$ • Additive identity = O
Multiplication	$A_{m \times k} \times B_{k \times q} = C_{m \times q}$ i.e., $\left[\sum_{r=1}^{k} a_{ir}\, b_{rj}\right] = [c_{ij}]$	• AB exist $\nRightarrow$ BA exists • AB may or may not be equal to BA • $(AB)\,C = A(BC)$ • $I_m \times A_{m \times m} = A_{m \times m} = A_{m \times m} \times I_m$ • $A\,(B + C) = AB + AC$, $(B + C)\,A = BA + CA$
Scalar Multiplication	$k\,A = B$ i.e., $k[a_{ij}]_{m \times n} = [k\,a_{ij}]_{m \times n} = [b_{ij}]_{m \times n}$	• $k\,(A + B) = kA + kB$ • $(k + m)A = kA + mA$

Comparable Matrices

Two matrices A and B are said to be comparable if they have same order *i.e.*, number of rows and columns of A are equal to number of rows and columns of B respectively.

MATRIX

A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

Equal Matrices

If matrices A and B are of same order, then $A = B$ iff $a_{ij} = b_{ij}\ \forall i, j$.

Types of Matrices

- Column Matrix : $A = [a_{ij}]_{m \times 1}$
- Row Matrix : $A = [a_{ij}]_{1 \times n}$
- Square Matrix : $A = [a_{ij}]_{m \times m}$
- Diagonal Matrix : $A = [a_{ij}]_{m \times m}$ where $a_{ij} = 0\ \forall\ i \neq j$
- Scalar Matrix : $A = [a_{ij}]_{n \times n}$ where $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ k, \text{ if } i = j \end{cases}$ for some constant k
- Zero Matrix : $A = [a_{ij}]$, where $a_{ij} = \{0\ \forall\ i \,\&\, j\}$
- Identity Matrix : $A = [a_{ij}]_{n \times n}$ where $a_{ij} = \begin{cases} 1, \text{ if } i = j \\ 0, \text{ if } i \neq j \end{cases}$
- Upper triangular matrix : A square matrix in which all the elements below the diagonal are zero.
- Lower triangular matrix : A square matrix in which all the elements above the diagonal are zero.
- Involutary Matrix : $A^2 = I$
- Orthogonal Matrix : $AA^T = A^T A = I$
- Idempotent Matrix : $A^2 = A$
- Unitary Matrix : $AA^\theta = A^\theta A = I$
- Symmetric Matrix : $A^T = A$
- Skew Symmetric Matrix : $A^T = -A$

Trace of a Matrix

The sum of the diagonal entries of a square matrix A is known as trace of matrix A. Let square matrix $A = \begin{pmatrix} a & 1 & 2 \\ 1 & b & 2 \\ 2 & 1 & c \end{pmatrix}$, then trace of matrix A is given by $a + b + c$.

Order of a Matrix

A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose of a Matrix

Transpose is obtained by interchanging rows and columns. If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ji}]_{n \times m}$

Properties

- $(A')' = A$
- $(kA)' = kA'$
- $(AB)' = B'A'$
- $(A \pm B)' = A' \pm B'$
- $(ABC)' = C'\,B'\,A'$

Inverse of a Matrix

If A and B are two square matrices such that $AB = BA = I$, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.

Properties

- Inverse of a square matrix, if it exists, is unique.
- $(A^{-1})^{-1} = A$
- $(kA)^{-1} = A^{-1}/k$
- $(AB)^{-1} = B^{-1} A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$

Elementary Operations

- $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
- $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$
- $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$

JEE 2022 PRACTICE PAPER ADVANCED

Exam on 28th August

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. If $f(x)=\int f(x)\,dx$, then $\int\left[f''(x)+\{f'(x)\}^2+\{f(x)\}^3\right]dx=0$ will give $\sum_{r=1}^{3}\frac{\{f(x)\}^r}{r}=$

(a) $f''(x)$ (b) constant
(c) $f^3(x)+c$ (d) None of these

2. If $a, b, c, d > 0$; $x \in R$ and $(a^2+b^2+c^2)x^2-2(ab+bc+cd)x+b^2+c^2+d^2 \le 0$, then $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$ is equal to

(a) 1 (b) −1 (c) 0 (d) 2

3. If z lies on the circle $|z-1|=1$, then $\frac{z-2}{z}$ equals

(a) 0 (b) 2
(c) −1 (d) none of these

4. If $2f(xy)=(f(x))^y+(f(y))^x\ \forall\ x, y$ and $f(1)=2$, then $\sum_{n=1}^{9} f(n)=$

(a) 1022 (b) 1023 (c) 1024 (d) 1025

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

Trigonometric equations which involve secant and cosecant are generally solved by converting them into their reciprocals cosine and sine. Care must be taken to avoid invalid solutions. Answer the following questions for the trigonometric equation $\sec\theta+\operatorname{cosec}\theta=c$ which is defined for $\theta \ne K\frac{\pi}{2}$, where $K \in I$

5. If $c=\sqrt{8}$ and $\theta \in (0, 2\pi)$, the no. of solutions of the equation is

6. If $c^2 < 8$ and $\theta \in (0, 2\pi)$, the no. of solutions of the equation is

Question Stem for Question Nos. 7 and 8

If $\cos\frac{\pi}{7}, \cos\frac{3\pi}{7}, \cos\frac{5\pi}{7}$ are the roots of the equation $8x^3-4x^2-4x+1=0$

7. The value of $\sec\frac{\pi}{7}+\sec\frac{3\pi}{7}+\sec\frac{5\pi}{7}$ is

8. The value of $\left(\sum_{r=1}^{3}\tan^2\left(\frac{2r-1}{7}\right)\right)\left(\sum_{r=1}^{3}\cot^2\left(\frac{2r-1}{7}\right)\right)$ is

Question Stem for Question Nos. 9 and 10

Let PQ be a focal chord of the parabola $y^2=4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y=2x+a$, $a>0$.

9. Length of the chord PQ is ma, then m equals

10. If chord PQ subtends an angle θ at the vertex of $y^2=4ax$, $\tan\theta=\frac{-2}{3}\sqrt{n}$, then n equals

SECTION 3

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*

11. Let $f(x)=\ln|x|$ and $g(x)=\sin x$. If A is the range of $f(g(x))$ and B is the range of $g(f(x))$, then

(a) $A\cup B=(-\infty, 1]$ (b) $A\cup B=(-\infty, \infty)$
(c) $A\cap B=[-1, 0]$ (d) $A\cap B=[0, 1]$

12. If $f(x)=\begin{cases}3x^2+12x-1, & -1\le x\le 2\\ 37-x, & 2<x\le 3\end{cases}$, Then

(a) $f(x)$ is increasing in $[-1,2]$

(b) $f(x)$ is continuous in $[-1,3]$

(c) $f'(2)$ does not exist

(d) $f(x)$ has maximum at $x = 2$

13. Tangents are drawn from a point P on the circle $C : x^2 + y^2 = a^2$ to the circle $C_1 : x^2 + y^2 = b^2$. If these tangents cut the circle C at Q and R and if QR is a tangent to the circle C_1, then the area of triangle PQR is

(a) $3\sqrt{3}\,b^2$ (b) $\frac{3\sqrt{3}}{4}a^2$

(c) $\frac{3\sqrt{3}}{2}ab$ (d) $2\sqrt{3}\,ab$

14. If $z_1 = 5 + 12i$ and $|z_2| = 4$, then

(a) maximum $(|z_1 + iz_2|) = 17$

(b) minimum $(|z_1 + (1 + i)z_2|) = 13-9\sqrt{2}$

(c) minimum $\left|\dfrac{z_1}{z_2+\dfrac{4}{z_2}}\right|=\dfrac{13}{4}$

(d) maximum $\left|\dfrac{z_1}{z_2+\dfrac{4}{z_2}}\right|=\dfrac{13}{3}$

15. If $(\sin^{-1}x + \sin^{-1}w)(\sin^{-1}y + \sin^{-1}z) = \pi^2$, then

$$D=\begin{vmatrix}x^{N_1} & y^{N_2}\\ z^{N_3} & w^{N_4}\end{vmatrix}(N_1,N_2,N_3,N_4\in N)$$

(a) has maximum value of 2

(b) has minimum value of 0

(c) has maximum value 1

(d) has minimum value of -2

16. If the plane $x + y + z = 1$ is rotated through 90° about its line of intersection with the plane $x - 2y + 3z = 0$, the new position of the plane is

(a) $x - 5y + 4z = 1$ (b) $x - 5y + 4z = -1$

(c) $x - 8y + 7z = 2$ (d) $x - 8y + 7z = -2$.

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is such that the length of tangents to it from the points (1, 0), (2, 0) and (3, 2) are respectively $1, \sqrt{7}$ and $\sqrt{2}$. Then $|c|$ is

18. If $I=\int_0^{\pi/2}\frac{x}{\sin x}dx$ and $J=\int_0^1\frac{\tan^{-1}x}{x}dx$, then $\frac{I}{J}$ is

19. Let $\vec{a},\vec{b},\vec{c}$ be coplanar unit vectors such that $\vec{b}\cdot\vec{c}=\cos\alpha$, $\vec{c}\cdot\vec{a}=\cos\beta, \vec{a}\cdot\vec{b}=\cos\gamma$. Then the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma$ is

PAPER-2

SECTION 1

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

1. Let $I_n=\int_{-\pi}^{\pi}\frac{\sin nx}{\sin x}dx,\ n\in N$ then

(a) $I_{n+2} = I_n$

(b) $\sum_{m=1}^{20} I_{2m+1}=20\pi$

(c) $I_{2m} = 0$ where $m = 1, 2, 3, \ldots$..

(d) $I_{n+1} = I_n$

2. The values of a for which

$\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}+\frac{a}{30}=0$ does not have a real solution is

(a) -10 (b) 12 (c) 5 (d) -30

3. If $a_1, a_2, a_3, \ldots, a_n$ is sequence of +ve numbers which are in A.P. with common difference 'd' and $a_1 + a_4 + a_7 + \ldots + a_{16} = 147$, then

(a) $a_1 + a_6 + a_{11} + a_{16} = 98$

(b) $a_1 + a_{16} = 49$

(c) $a_1 + a_4 + a_7 + \ldots + a_{16} = 6a_1 + 45d$

(d) Maximum value of $a_1a_2 \ldots a_{16}$ is $\left(\frac{49}{2}\right)^{16}$

4. Let $f(x)=\begin{cases} 2x-4 & x\le 2 \\ -x^2+\dfrac{k^3(k-1)^2}{k^2-k-2}+4, & x>2 \end{cases}$.

$f(x)$ attains local maximum at $x = 2$ if k lies in

(a) $(0, 1)$ (b) $(3, \infty)$

(c) $(-\infty, -1)$ (d) $(1, 2)$

5. If a right angled ΔABC of maximum area is inscribed within a circle of radius R, then (Δ represents area of triangle ABC and r, r_1, r_2, r_3 represent inradius and exradii, and s is the semi perimeter of ΔABC, then

(a) $\Delta = R^2$ (b) $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{\sqrt{2}+1}{R}$

(c) $r=(\sqrt{2}-1)R$ (d) $s=(1+\sqrt{2})R$

6. Find the value of $\lim_{x\to 0}\left(\int_0^1 (by+a(1-y))^x\,dy\right)^{\frac{1}{x}}$ where $b > a$.

(a) $\frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}}$ (b) $\frac{b^a}{a^a}$

(c) $\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}}$ (d) $e\left(\frac{b^b}{a^a}\right)$

SECTION 2

- ***This section contains THREE (03) question stems.***
- ***There are TWO (02) questions corresponding to each question stem.***
- ***The answer to each question is a NUMERICAL VALUE.***

Question Stem for Question Nos. 7 and 8

Let the curves $S_1 : y = x^2$, $S_2 : y = -x^2$, $S_3 : y^2 = 4x - 3$

7. Area bounded by the curves S_1, S_2, S_3 is

8. Area bounded by the curves S_1, S_3 and the line $x = 3$ is

Question Stem for Question Nos. 9 and 10

Let each of the circles

$S_1 \equiv x^2 + y^2 + 4y - 1= 0$

$S_2 \equiv x^2 + y^2 + 6x + y + 8 = 0$

$S_3 \equiv x^2 + y^2 - 4x - 4y - 37 = 0$

touches the other two. Let P_1, P_2, P_3 be the point of contact of S_1 and S_2, S_2 and S_3, S_3 and S_1 respectively. Let T be the point of concurrence of the tangents at P_1, P_2, P_3 to the circles. C_1, C_2, C_3 are the centres of S_1, S_2, S_3 respectively.

9. The area of the quadrilateral $TP_2C_3P_3$ is

10. The ratio of the area of $\frac{\Delta P_1P_2P_3}{\Delta C_1C_2C_3}$ is $\frac{m}{n}$, then $m + n =$

Question Stem for Question Nos. 11 and 12

If the curves $y = f(x)$ and $y = g(x)$ intersects at n different points then $f(x) = g(x)$ is said to have 'n' solutions.

11. Number of solutions of $|\cos x| = 2[x]$ (where [ ] is step function) is

12. The number of solutions of $\sin\pi x = |\log_e|x||$ is

SECTION 3

- ***This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***

Paragraph for Q. 13 and Q. 14

Let L_1 and L_2 be the lines $x + 2y - z - 3 = 0 = 3x - y + 2z - 1$ and $2x - 2y + 3z - 2 = 0 = x - y + z + 1$.

13. The distance of the origin from the point of intersection of L_1 and L_2 is

(a) $\sqrt{22}$ (b) $\sqrt{33}$ (c) $2\sqrt{11}$ (d) $\sqrt{11}$

14. The distance of the origin from the plane through the lines is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{4\sqrt{2}}$

Paragraph for Q. 15 and Q. 16

An equation of the form $2m\log_a f(x) = \log_a g(x)$, $a > 0$, $a \ne 1$, $m \in N$ is equivalent to the system

15. The no. of solutions of $2\log 2x = \log_e(7x - 2 - 2x^2)$ is

(a) 1 (b) 2 (c) 3 (d) Infinite

16. The solution set of the equation $\log_{(x^3+6)}(x^2-1) = \log_{(2x^2+5x)}(x^2-1)$ is

(a) $\{-2\}$ (b) $\{1\}$ (c) $\{3\}$ (d) $\{-2, 1, 3\}$

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$ where $i = \sqrt{-1}$. The value of the expression

$$\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|} \text{ is}$$

18. $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & x < \dfrac{\pi}{2} \\ a \quad, & x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi - 2x)^2}, & x > \dfrac{\pi}{2} \end{cases}$

If $f(x)$ is continuous at $x = \pi/2$, then $\frac{b}{a} =$

19. If $I = \int\limits_{-\pi/2}^{2\pi} \sin^{-1}(\sin x)\,dx$, then $-\frac{16}{\pi^2} I =$

SOLUTIONS

PAPER-1

1. (b) : Since, $f(x) = \int f(x)dx$

$\therefore\ f'(x) = f(x)$ ...(i)

$\therefore\ \int [f''(x) + \{f'(x)\}^2 + \{f(x)\}]^3 dx = 0$

$\Rightarrow\ f'(x) + \int f(x)\, f'(x)\, dx + \int \{f(x)\}^2 \cdot f(x)\, dx = 0$

$\Rightarrow\ f'(x) + \frac{\{f(x)\}^2}{2} + \int \{f(x)\}^2 f'(x)\, dx = 0$

$\Rightarrow\ f'(x) + \frac{\{f(x)\}^2}{2} + \frac{\{f(x)\}^3}{3} + C = 0$

$\therefore\ \frac{f(x)}{1} + \frac{\{f(x)\}^2}{2} + \frac{\{f(x)\}^3}{3} = -C$, which is constant.

2. (c) : $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$

$\Rightarrow\ (ax - b)^2 + (bx - c)^2 + (cx - d)^2 \le 0$

$\Rightarrow\ (ax - b)^2 + (bx - c)^2 + (cx - d)^2 = 0$

$\Rightarrow\ \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = x\ \Rightarrow b^2 = ac$

or $2\log b = \log a + \log c$ ....(i)

Now, $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = \begin{vmatrix} 130 & 54 & \log a + \log c \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$

Applying $R_1 \to R_1 + R_3$

$$= 2\begin{vmatrix} 65 & 27 & \log b \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = 0 \qquad \text{(Using (i))}$$

3. (d) **4. (a)**

5. (1)

6. (2) : Let $\tan\theta = x$

$\sqrt{x^2+1} + \sqrt{\frac{1}{x^2}+1} = c \Rightarrow x^2 + x + 1 = \pm x\sqrt{c^2+1}$ ... (1)

If $c = \sqrt{8} \Rightarrow x^2 + x + 1 = \pm 3x$

$\Rightarrow\ x^2 - 2x + 1 = 0$ or $x^2 + 4x + 1 = 0$

$\Rightarrow\ x = 1$ or $-2 \pm \sqrt{3}$

$\tan\theta = 1$ or $\tan\theta = -2 \pm \sqrt{3} < 0$ (Impossible)

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$, But $\theta = \frac{5\pi}{4}$ (not possible)

From (1), $x^2 + (1 + \sqrt{c^2+1})x + 1 = 0$

or $x^2 + (1 - \sqrt{c^2+1})x + 1 = 0$

The equation has real solutions if $(1 + \sqrt{c^2+1})^2 - 4 \ge 0$ and $(1 - \sqrt{c^2+1})^2 - 4 \ge 0$

$\Rightarrow\ (3 - \sqrt{c^2+1}) \le 0$ and

$(\sqrt{c^2+1} + 3)(\sqrt{c^2+1} - 1) \ge 0$

The 2nd inequality holds for all c^2 while the 1st inequality hold if $c^2 \ge 8$

If $c^2 < 8$ then, the equation has 2 solutions in $(0, 2\pi)$

If $c^2 > 8$ then, the equation has 4 solutions in $(0, 2\pi)$

7. (4) **8. (105)**

$8x^3 - 4x^2 - 4x + 1 = 0$ ... (1)

$x \to \frac{1}{x} \Rightarrow x^3 - 4x^2 - 4x + 8 = 0$ ... (2)

$\sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7} = 4$

$\Rightarrow\ x(x^2 - 4) = 4(x^2 - 2)$

$\Rightarrow\ \sqrt{x+1}(x + 1 - 4) = 4(x + 1 - 2)$, Replacing $x^2 \to x+1$ in (2)

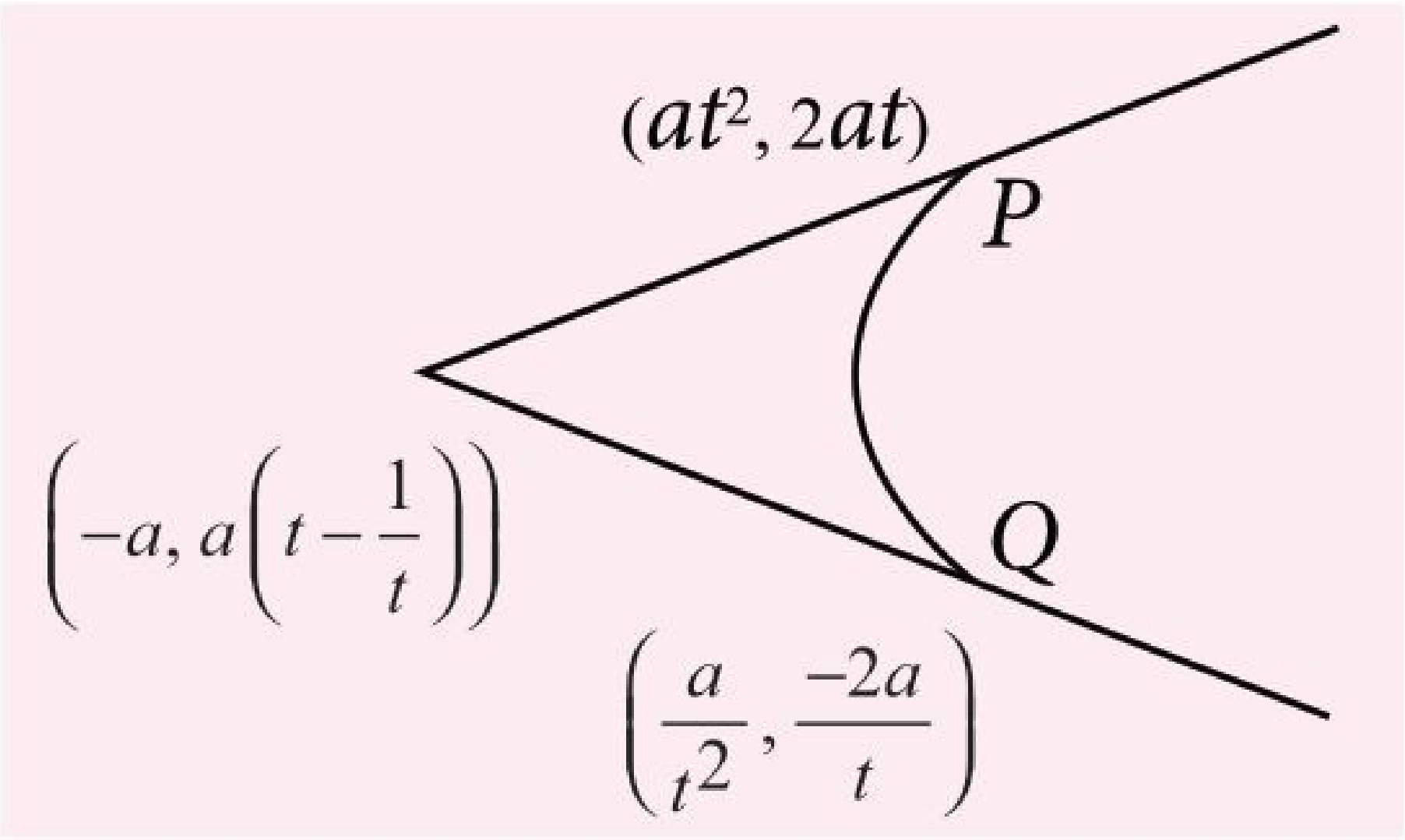
$\Rightarrow\ x^3 - 21x^2 + 35x - 7 = 0$ ... (3)

$x \to \frac{1}{x}$ in (3) $\Rightarrow 7x^3 - 35x^2 + 21x - 1 = 0$

$$\cot^2\frac{\pi}{7}+\cot^2\frac{3\pi}{7}+\cot^2\frac{5\pi}{7}=\frac{35}{7}=5$$

$$\left(\sum_{r=1}^{3}\tan^2\left(\frac{2r-1}{7}\right)\right)\left(\sum_{r=1}^{3}\cot^2\left(\frac{2r-1}{7}\right)\right)=21\times5=105$$

9. (5) :



$(at^2, 2at)$
P
$\left(-a, a\left(t-\frac{1}{t}\right)\right)$
Q
$\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

Given $a\left(t-\frac{1}{t}\right)=-a.\quad \therefore\ t-\frac{1}{t}=-1$

$$PQ=a\left(t+\frac{1}{t}\right)^2=a\left\{\left(t-\frac{1}{t}\right)^2+4\right\}=5a$$

10. (5) : As $t-\frac{1}{t}=-1$, we have $t+\frac{1}{t}=\sqrt{5}$

The angle θ between them will be given by

$$\tan\theta=\frac{\frac{2}{t}+2t}{1-4}=\frac{2\left(t+\frac{1}{t}\right)}{-3}=-\frac{2\sqrt{5}}{3}$$

11. (a, c) : The range of $|\sin x| = [0, 1]$.

A = range of $\ln|\sin x| = (-\infty, 0]$.

The range of $\ln|x| = (-\infty, \infty)$.

B = range of $\sin(\ln|x|) = [-1, 1]$

$\therefore\ A \cup B = (-\infty, 0] \cup [-1, 1] = (-\infty, 1]$;

$A \cap B = (-\infty, 0) \cap [-1, 1] = [-1, 0]$.

12. (a,b,c,d) : $f(2^-) = f(2) = f(2^+) = 35$

$\therefore\ f(x)$ is continuous, $f'(x) = \begin{cases}6x+12, -1\le x<2\\ -1, 2<x\le 3\end{cases}$

$f'(2^-) = 24,\ f'(2^+) = -1,\ \therefore\ f'(2)$ does not exist.

$f'(x) = 6(x+2) > 0$ in $[-1, 2]$, $\therefore\ f(x)$ increases in $[-1, 2]$

$\therefore\ f(x)$ has maximum value 35 at $x = 2$.

13. (a, b, c)

14. (a, d) : $z_1 = 5 + 12i,\ |z_2| = 4$

$|z_1+iz_2| \le |z_1|+|z_2| = 13+4=17$

$\therefore\ |z_1+(1+i)z_2| \ge ||z_1|-|1+i||z_2|| = 13-4\sqrt{2}$

$\therefore\ \min(|z_1+(1+i)z_2|) = 13-4\sqrt{2}$

$$\left|z_2+\frac{4}{z_2}\right|\le|z_2|+\frac{4}{|z_2|}=4+1=5$$

$$\left|z_2+\frac{4}{z_2}\right|\ge|z_2|-\frac{4}{|z_2|}=4-1=3$$

$$\therefore\ \max\left|\frac{z_1}{z_2+\frac{4}{z_2}}\right|=\frac{13}{3}\text{ and }\min\left|\frac{z_1}{z_2+\frac{4}{z_2}}\right|=\frac{13}{5}$$

15. (a, d) : $(\sin^{-1}x + \sin^{-1}w)(\sin^{-1}y + \sin^{-1}z) = \pi^2$

$\Rightarrow\ \sin^{-1}x + \sin^{-1}w = \sin^{-1}y + \sin^{-1}z = \pi$

or, $\sin^{-1}x + \sin^{-1}w = \sin^{-1}y + \sin^{-1}z = -\pi$

$\Rightarrow\ x = y = z = w = 1$ or, $x = y = z = w = -1$

Hence, maximum value of $\begin{vmatrix}x^{N_1} & y^{N_2}\\ z^{N_3} & w^{N_4}\end{vmatrix}=\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}=2$

and minimum value $=\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}=-2$

16. (d) **17. (3)**

18. (2)

19. (1) : $[\vec{a}\ \vec{b}\ \vec{c}]=0 \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}]^2=0$

$$\Rightarrow \begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}=\begin{vmatrix}1 & \cos\gamma & \cos\beta\\ \cos\gamma & 1 & \cos\alpha\\ \cos\beta & \cos\alpha & 1\end{vmatrix}=0$$

$\Rightarrow\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma = 1.$

PAPER-2

1. (a, c) : $I_n = 2\int_0^{\pi}\frac{\sin nx}{\sin x}dx$

$\left(\because f(x)=\frac{\sin nx}{\sin x}\text{ is an even function}\right)$

$\Rightarrow\ I_{n+2} - I_n = 0$

2. (b, c, d) : $\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}=\frac{(x-1)(x-2)(x-3)}{(x-1)(x-2)(x+4)}$

$\therefore\ x \ne 1, 2, -4$ then $f(x)=\frac{x-3}{x+4}$

Range of $f(x)=R-\left\{1,-\frac{2}{5},-\frac{1}{6}\right\}$

So equation does not have a solution if $\frac{a}{30}=-1,\frac{2}{5},\frac{1}{6}$

$\Rightarrow\ a = -30, 12, 5$

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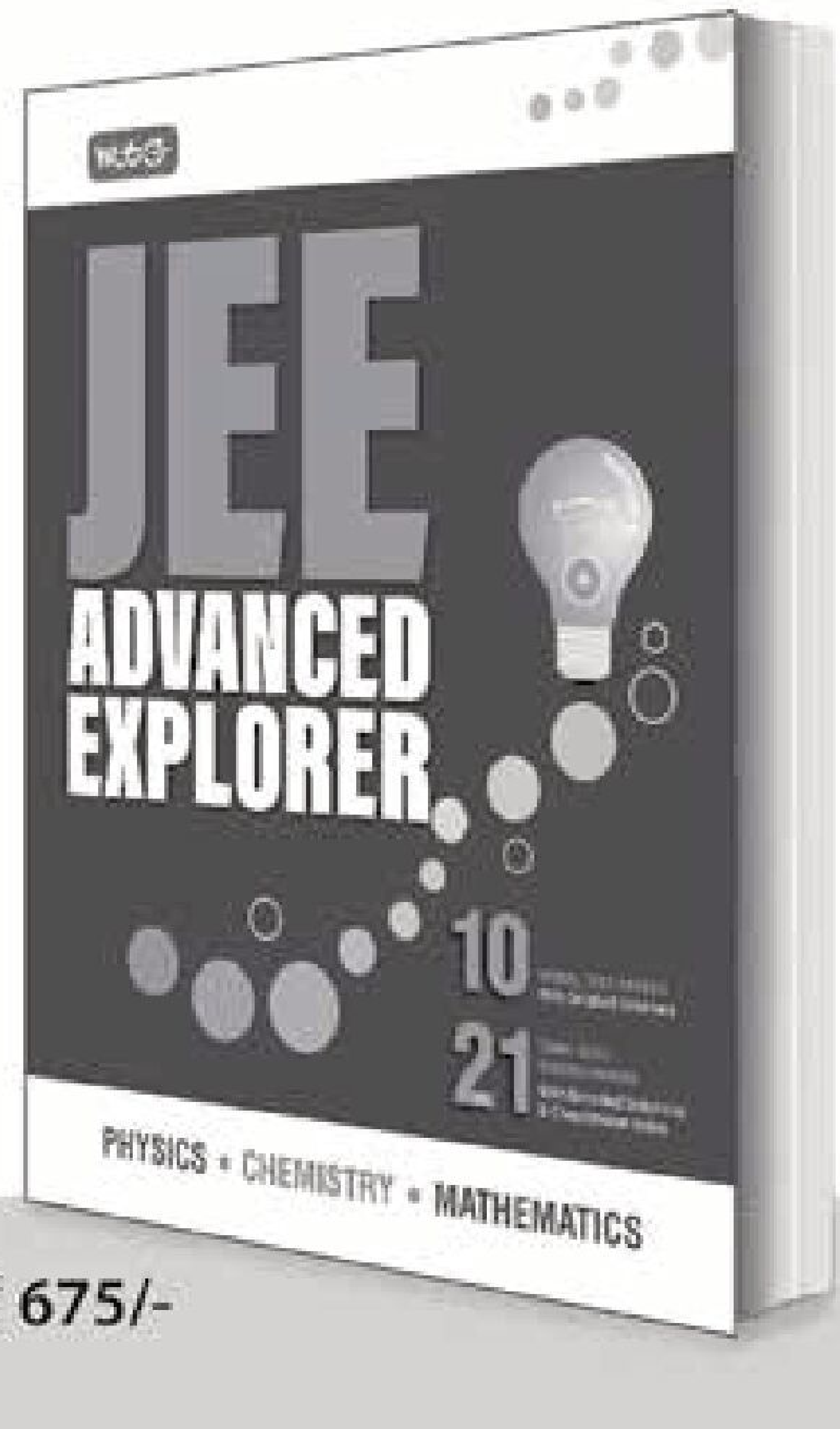
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3. (a, b, c, d) : $a_1 + a_4 + a_7 + \ldots + a_{16} = 147$

$\Rightarrow\ 3(a_1 + a_{16}) = 147 \quad \Rightarrow\ a_1 + a_{16} = 49$

Again $a_1 + a_4 + a_7 + a_{10} + \ldots + a_{16}$

$= a_1 + a_1 + 3d + a_1 + 6d + \ldots + a_1 + 15d$

$= 6a_1 + 45d = 147 \quad \Rightarrow\ 2a_1 + 15d = 49$

$a_1 + a_6 + a_{11} + a_{16} = a_1 + a_1 + 5d + a_1 + 10d + a_1 + 15d$

$= 4a_1 + 30d = 2(2a_1 + 15d) = 2(49) = 98$

Now using A.M. ≥ G.M.

$$\frac{a_1 + a_2 + \ldots + a_{16}}{16} \geq (a_1 a_2 a_3 \ldots a_{16})^{\frac{1}{16}}$$

$$\Rightarrow \frac{8(a_1 + a_{16})}{16} \geq (a_1 a_2 a_3 \ldots a_{16})^{1/16}$$

$$\Rightarrow \left(\frac{49}{2}\right)^{16} \geq a_1 a_2 a_3 \ldots a_{16}$$

4. (a, c, d) : When $f(x)$ is continuous at $x = 2$

$f'(x)$ does not exist at $x = 2$ and $f'(x)$ changes sign from + to –

$\Rightarrow\ f(x)$ attains max. at $x = 2$ if

$$\frac{k^3(k-1)^2}{k^2 - k - 2} = 0 \ \Rightarrow\ k = 0, 1$$

When $f(x)$ is discontinuous at $x = 2$, $f'(x)$ changes its sign from + to –. $f(x)$ will attain maximum if $\lim\limits_{x\to 2^+} f(x) < f(2)$ as

$\lim\limits_{x\to 2^-} f(x) = f(2)$ i.e if $k \in (-\infty, -1) \cup (0, 1) \cup (1, 2)$

$\Rightarrow\ k \in (-\infty, -1) \cup [0, 2)$

5. (a, b, c, d) : For a right angled triangle inscribed in a circle of radius R, the length of the hypotenuse is $2R$. Then area is maximum when it is an isosceles triangle with each side $= \sqrt{2}R$

$$\therefore\ s = \frac{1}{2}(2\sqrt{2} + 2)R = (\sqrt{2} + 1)R$$

$$\Delta = \frac{1}{2}\sqrt{2}R \cdot \sqrt{2}R = R^2$$

$$r = \frac{\Delta}{s} = \frac{R^2}{(\sqrt{2}+1)R} \ \Rightarrow\ r = (\sqrt{2} - 1)R$$

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r} = \frac{1}{(\sqrt{2}-1)R} = \frac{\sqrt{2}+1}{R}$$

6. (a) : Let $L = \lim\limits_{x\to 0}\left(\int\limits_0^1 (by + a(1-y))^x dy\right)^{\frac{1}{x}}$

Let $I = \int_0^1 (by + a(1-y))^x\, dy$

Let $by + a(1-y) = t$, $(b - a)\, dy = dt$

$$\Rightarrow\ I = \int \frac{t^x}{(b-a)} dt = \frac{t^{x+1}}{(x+1)(b-a)}$$

$$I = \left[\frac{(by + a(1-y))^{x+1}}{(x+1)(b-a)}\right] \text{[computed at } y = 1 \text{ and } y = 0]$$

$$I = \frac{b^{x+1} - a^{x+1}}{(x+1)(b-a)}$$

$$\Rightarrow\ L = \lim_{x\to 0}\left(\frac{b^{x+1} - a^{x+1}}{(x+1)(b-a)}\right)^{\frac{1}{x}}$$

$$ln(L) = \lim_{x\to 0}\frac{ln\left(\frac{(b^{x+1} - a^{x+1})}{(x+1)(b-a)}\right)}{x}$$

As x approaches 0, denominator and numerator approaches 0.

Hence, we can use L-hospital's rule.

MtG

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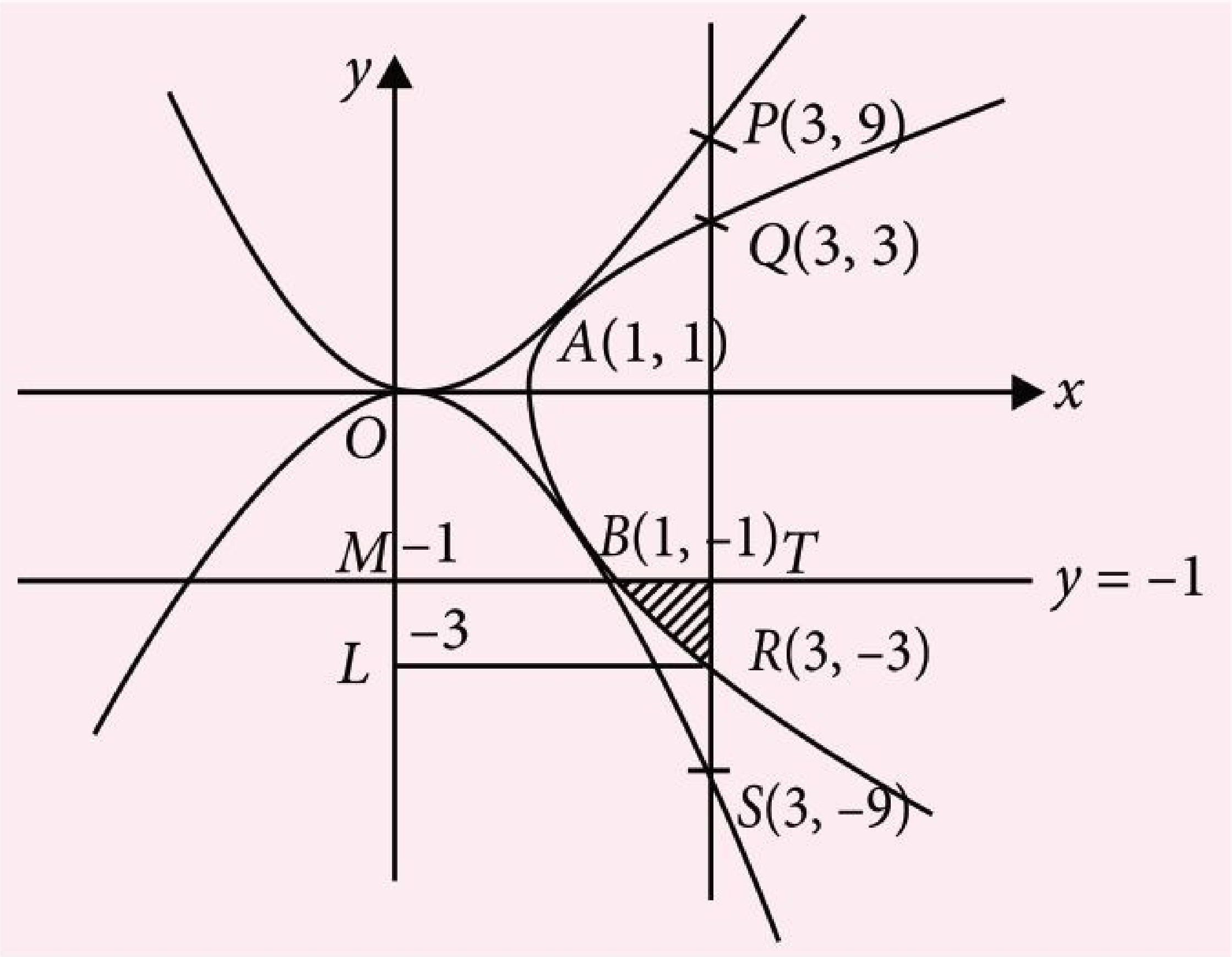
$$ln(L)=\lim_{x\to 0}\left(\frac{b^{x+1}ln(b)-a^{x+1}ln(a)}{(x+1)(b-a)}-\frac{1}{x+1}\right)$$

$$ln(L)=\frac{1}{b-a}ln\frac{b^b}{a^a}-1 \Rightarrow L=\frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{(b-a)}}$$

7. (0.33) :



Area $OAB=2\int_0^1\left(\frac{y^2+3}{4}-\sqrt{y}\right)dy=\frac{1}{3}$

8. (4.33) : Area $APQ=\int_1^3(x^2-\sqrt{(4x-3)})dx = \frac{13}{3}$

9. (15) **10. (7)**

(11 - 12) :

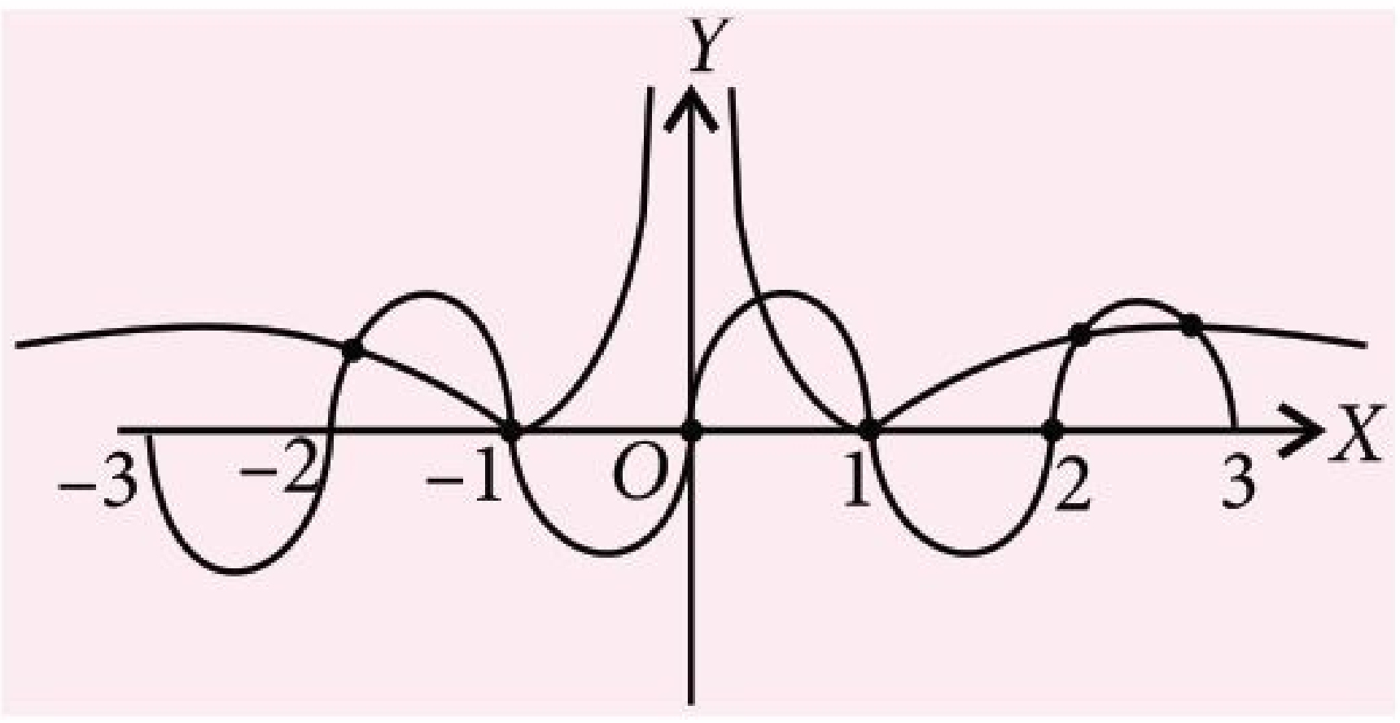
11. (0) : As $0 \le |\cos x| \le 1$, we have $0 \le 2[x] \le 1$

$\Rightarrow \ [x] = 0 \ \Rightarrow \ 0 \le x < 1$

$|\cos x|=0 \ \Rightarrow x=2n\pi+\frac{\pi}{2}$

No solution in [0, 1).

12. (6) : No of solutions = 6



13. (b) : L_1 is $x + 2y = 3 + z$, $3x - y = 1 - 2z$ ...(i)

L_2 is $2x - 2y = 2 - 3z$, $x - y = -1 - z$ ...(ii)

(ii) $\Rightarrow 2 - 3z = -2 - 2z \Rightarrow z = 4$

$x + 2y = 7$, $2x - 2y = -10 \Rightarrow x = -1, y = 4, z = 4$

The distance of the point (–1, 4, 4) from the origin is $\sqrt{1+16+16}=\sqrt{33}$.

14. (c) : The plane through L_1 is

$x + 2y - z - 3 + \lambda(3x - y + 2z - 1) = 0$ ...(i)

The plane through L_2 is

$2x - 2y + 3z - 2 + \mu(x - y + z + 1) = 0$ ...(ii)

(i) and (ii) are same if $\frac{1+3\lambda}{2+\mu}=\frac{2-\lambda}{-(2+\mu)}$

$\Rightarrow \ 1+3\lambda=\lambda-2 \ \Rightarrow \ \lambda=\frac{-3}{2}$

Now (i) gives $7x - 7y + 8z + 3 = 0$

The distance of (0, 0, 0) from it is $\frac{3}{\sqrt{49+49+64}}$ $=\frac{3}{\sqrt{162}}=\frac{1}{3\sqrt{2}}.$

15. (b): $\begin{cases}2x>0\\(2x)^2=7x-2-2x^2\end{cases} \Rightarrow \begin{cases}x>0\\6x^2-7x+2=0\end{cases}$

$\Rightarrow \begin{cases}x>0\\x=\frac{1}{2},\frac{2}{3}\end{cases}$

16. (c) : $\begin{cases}x^2-1>0\\2x^2+5x>0, x\ne 1 \Rightarrow x<\frac{-5}{2} \text{ and } x>1\\x^3+6=2x^2+5x\end{cases}$

17. (4) : $\alpha_k=\cos\frac{2k\pi}{14}+i\sin\frac{2k\pi}{14}=e^{i\frac{2\pi k}{14}}$

$$\frac{\sum_{k=1}^{12}\left|e^{i2(k+1)\frac{\pi}{14}}-e^{i\frac{2k\pi}{14}}\right|}{\sum_{k=1}^{3}\left|e^{\frac{i2(4k-1)\pi}{14}}-e^{i\frac{2(4k-2)\pi}{14}}\right|}=\frac{\sum_{k=1}^{12}\left|e^{i\frac{2\pi}{14}}-1\right|}{\sum_{k=1}^{3}\left|e^{i\frac{2\pi}{14}-1}\right|}=\frac{12}{3}=4$$

18. (8) : L.H.L. $=\lim_{x\to\frac{\pi}{2}^-}f(x)=\frac{1}{2}$

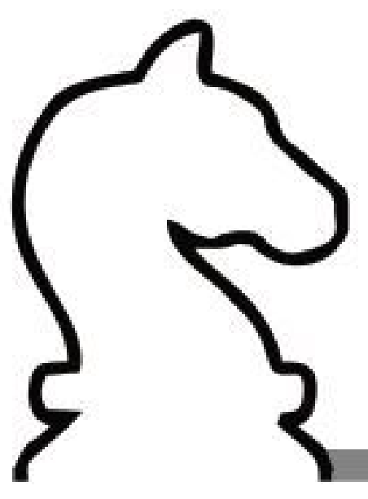
and R.H.L. $=\lim_{x\to\frac{\pi}{2}^+}f(x)=\frac{b}{8}$

$\therefore \ \frac{1}{2}=a=\frac{b}{8}$

19. (2) : $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\,dx+\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(\pi-x)\,dx+\int_{\frac{3\pi}{2}}^{2\pi}(x-2\pi)\,dx=\frac{-\pi^2}{8}$

◆ ◆

Challenging PROBLEMS ON JEE



MULTIPLE CORRECT ANSWER TYPE

1. If $f(x) = \cos(ix) - i\sin(ix)$ then $\frac{2}{\underline{|1}} + \frac{6}{\underline{|2}} + \frac{12}{\underline{|3}} + \ldots$ to $\infty =$

(a) $3f(1) - 1$ (b) $3f(1)$

(c) $f(1)$ (d) none of these

2. Find the number of positive solution of $400x^5 + x^3 - 3x^2 + 16x = 2013$.

(a) 0 (b) 1

(c) 5 (d) none of these

3. $\dfrac{\sum_{r=0}^{n-1}\pi^r}{\sum_{r=0}^{n}\pi^r} - \dfrac{\sum_{r=1}^{n}e^{r-1}}{\sum_{r=1}^{n+1}e^{r-1}}$ is

(a) > 0 (b) < 0

(c) $= 0$ (d) ≤ 0

4. If $A_1, A_2, A_3, \ldots, A_{1006}$ be independent events so that $P(A_i) = \frac{1}{2i}$ $(i = 1, 2, 3, \ldots, 1006)$ and probability that none of the events occurs be $\dfrac{\underline{|\alpha}}{2^{\alpha}(\underline{|\beta})^2}$ then

(a) $\alpha = \beta$ (b) $\alpha = 2\beta$

(c) β is not a prime no. (d) $\beta^2 = 2\alpha$

5. If θ and $2^n\theta$ be supplementary to each other, then $\prod_{r=0}^{n-1}(1 + 2\cos 2^r\theta) \geq$

(a) 2^n (b) $2^{n/2}$

(c) 1 (d) none of these

6. In ΔABC, $\prod\left(1 + \frac{b-c}{a}\right)^a$ always belongs to

(a) $(0, 2)$ (b) $(0, 1]$

(c) $\left[\frac{1}{2}, 1\right]$ (d) none of these

7. If $f(\theta) = \cot\theta - \cot^2\theta + \cot^3\theta - \ldots$ to ∞; $\alpha + \beta = \frac{5\pi}{4}$, then

(a) $f(\alpha)f(\beta) = \frac{1}{2}$ always

(b) $f\left(\frac{2\pi}{3}\right)f\left(\frac{7\pi}{12}\right) = f\left(\frac{\pi}{4}\right)$

(c) $f\left(\frac{\pi}{3}\right)f\left(\frac{11\pi}{12}\right) = \frac{1}{2}$

(d) $f\left(\frac{5\pi}{12}\right)f\left(\frac{5\pi}{6}\right) = f\left(\frac{\pi}{4}\right)$

8. If for $n > 1, a_{n-1}(a_n - a_{n-1}) = 1, \lim_{x\to 0^+}\{1 + (\sin x)^{1/x}\} = a_1$ and $a_{403}^2 \in (p, q)$, then $p + q =$

(a) 2013 (b) 1212

(c) 2017 (d) none of these

9. If tangent to $25y^2 - 16x^2 = 400$ at (h, k) meets $x^2 + y^2 = 16$ at $P(x_1, y_1)$ and $Q(x_2, y_2)$, then which are in H.P. ?

(a) x_1, h, x_2 (b) x_1, k, x_2

(c) $x_1^{-1}, h^{-1}, x_2^{-1}$ (d) $x_1^{-1}, k^{-1}, x_2^{-1}$

10. If $t_0, t_1, t_2, \ldots, t_{100}$ be the terms in the expansion of $(x + a)^{100}$, then

$$\int \frac{2x\,dx}{\left(\sum_{r=0}^{50}(-1)^r t_{2r}\right)^2 + \left(\sum_{r=0}^{49}(-1)^r t_{2r+1}\right)^2} =$$

(a) $C - \frac{1}{99(x^2 + a^2)^{99}}$ (b) $C + \frac{1}{100(x^2 + a^2)^{100}}$

(c) $C - 99(x^2 + a^2)^{99}$ (d) none of these

11. If a be the digit at unit's place in $11^{2012} + 23^{2012} - 3^{2012}$, then $\int_{a-1}^{a}\frac{dx}{\sqrt{1 - x^2} - x + \frac{1}{x}} =$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

12. The solution of $(ax + h)dx = (by + k)dy$ represents a parabola, then

(a) at least one of a and b is zero
(b) $a \neq 0$, $b \neq 0$
(c) both a and b are zero
(d) only one of a and b is zero

13. If z_1, z_2 and z_3 be the vertices of ΔABC, taken in anti-clock wise direction and z_0 be the circumcentre, then $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to

(a) 0 (b) 1 (c) – 1 (d) 2

14. Let $A(\vec{a})$ and $B(\vec{b})$ be points on two skew lines $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ and the shortest distance between the skew lines is 1, where $\vec{p}$ and $\vec{q}$ are unit vectors forming adjacent sides of a parallelogram enclosing an area of 1/2 units. If an angle between AB and the line of shortest distance is 60°, then AB =

(a) 1/2 (b) 2
(c) 1 (d) $\lambda \in R - \{0\}$

15. Let $x \in \left(\frac{5\pi}{2}, 3\pi\right)$ then

(a) $\cos^{-1}(\sin(\cos^{-1}(\cos x) - 2\tan^{-1}(\tan x))) = \frac{7\pi}{2} - x$

(b) $\cos^{-1}(\sin(\cos^{-1}(\cos x) - 2\tan^{-1}(\tan x))) = \frac{9\pi}{2} - x$

(c) $\tan^{-1}(\cot(2\sin^{-1}(\sin x) - \tan^{-1}(\tan x))) = \frac{\pi}{2} + x$

(d) $\tan^{-1}(\cot(2\sin^{-1}(\sin x) + \tan^{-1}(\tan x))) = x - \frac{5\pi}{2}$

SOLUTIONS

1. (b) : $f(x) = e^{i(-ix)}$ [$\because \cos\theta + i\sin\theta = e^{i\theta}$] $= e^x$

Now, $t_r = \frac{r^2 + r}{\underline{|r}} = \frac{r+1}{\underline{|r-1}} = \frac{(r-1)+2}{\underline{|r-1}} = \frac{1}{\underline{|r-2}} + \frac{2}{\underline{|r-1}}$

$\therefore \sum_{r=1}^{\infty} t_r = \sum_{r=1}^{\infty} \frac{1}{\underline{|r-2}} + \sum_{r=1}^{\infty} \frac{2}{\underline{|r-1}}$

$\therefore$ $t_1 + t_2 + t_3 + ...$ to ∞

$= \left(\frac{1}{\underline{|-1}} + \frac{1}{\underline{|0}} + \frac{1}{\underline{|1}} + ...\text{to } \infty\right) + 2\left(\frac{1}{\underline{|0}} + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + ...\text{to } \infty\right)$

$= e + 2e\left[\because \frac{1}{\underline{|-1}} = 0 \text{ etc}\right] = 3e = 3f(1)$

2. (b) : Let $f(x) = 400x^5 + x^3 - 3x^2 + 16x - 2013$

$f'(x) = 2000x^4 + 3x^2 - 6x + 16$
$= 2000x^4 + 3(x^2 - 2x + 1) + 13$
$= 2000x^4 + 3(x - 1)^2 + 13$

$\therefore$ $f'(x) > 0$ for all real x
$\Rightarrow$ $f(x)$ is an increasing function.
Also, $f(-\infty) = -\infty$, $f(0) = -2013$, $f(\infty) = \infty$
$\therefore$ $f(x) = 0$ will cross x-axis once for some $x > 0$
$\therefore$ Number of positive solution of $f(x) = 0$
i.e. $400x^5 + x^3 - 3x^2 + 16x = 2013$ is 1.

3. (b) : $\because \pi > e \therefore \frac{1}{\pi} < \frac{1}{e}, \frac{1}{\pi^2} < \frac{1}{e^2}, ...$ etc.

$$\therefore \frac{1}{\pi} + \frac{1}{\pi^2} + ... + \frac{1}{\pi^n} < \frac{1}{e} + \frac{1}{e^2} + ... + \frac{1}{e^n}$$

$$\Rightarrow \frac{1}{\frac{1}{\pi} + \frac{1}{\pi^2} + ... + \frac{1}{\pi^n}} > \frac{1}{\frac{1}{e} + \frac{1}{e^2} + ... + \frac{1}{e^n}}$$

$$\Rightarrow \frac{\pi^n}{\pi^n\left(\frac{1}{\pi} + \frac{1}{\pi^2} + ... + \frac{1}{\pi^n}\right)} > \frac{e^n}{e^n\left(\frac{1}{e} + \frac{1}{e^2} + ... + \frac{1}{e^n}\right)}$$

$$\Rightarrow \frac{\pi^n}{\pi^{n-1} + \pi^{n-2} + ... + \pi + 1} > \frac{e^n}{e^{n-1} + e^{n-2} + ... + e + 1}$$

$$\Rightarrow \frac{\pi^n}{1 + \pi + ... + \pi^{n-1}} + 1 > \frac{e^n}{1 + e + ... + e^{n-1}} + 1$$

$$\Rightarrow \frac{1 + \pi + ... + \pi^n}{1 + \pi + ... + \pi^{n-1}} > \frac{1 + e + ... + e^n}{1 + e + ... + e^{n-1}}$$

$$\Rightarrow \frac{1 + \pi + ... + \pi^{n-1}}{1 + \pi + ... + \pi^n} < \frac{1 + e + ... + e^{n-1}}{1 + e + ... + e^n}$$

$$\Rightarrow \frac{\sum_{r=0}^{n-1} \pi^r}{\sum_{r=0}^{n} \pi^r} - \frac{\sum_{r=1}^{n} e^{r-1}}{\sum_{r=1}^{n+1} e^{r-1}} < 0$$

4. (b, c) : Here, $P(A_1) = \frac{1}{2}, P(A_2) = \frac{1}{4}$,

$P(A_3) = \frac{1}{6}, ..., P(A_{1006}) = \frac{1}{2012}$

$\therefore$ Probability that none of the independent events $A_1, A_2, A_3, ...$ occurs

$= P(A_1')P(A_2')P(A_3').....P(A_{1006}')$

$= \{1 - P(A_1)\}\{1 - P(A_2)\}\{1 - P(A_3)\}.......\{1 - P(A_{1006})\}$

$= \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{6}\right)....\left(1 - \frac{1}{2012}\right)$

$= \frac{1 \cdot 3 \cdot 5 ..... 2011}{2 \cdot 4 \cdot 6 ..... 2012}$

$$=\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\,.....\,2011\cdot 2012}{(2\cdot 4\cdot 6\,.....\,2012)(2\cdot 4\cdot 6\,.....\,2012)}$$

$$=\frac{\underline{|2012}}{2^{2012}(1\cdot 2\cdot 3\,.....\,1006)^2}=\frac{\underline{|2012}}{2^{2012}(\underline{|1006})^2}=\frac{\underline{|\alpha}}{2^{\alpha}\cdot(\underline{|\beta})^2}$$

(given)

∴ $\alpha = 2012,\ \beta = 1006$

5. (a) : ∵ θ and $2^n\theta$ are supplementary to each other

∴ $\theta + 2^n\theta = \pi \Rightarrow 2^n\theta = \pi - \theta$ ... (1)

∵ $\cos\theta\cos2\theta\cos 2^2\theta\cos2^3\theta\ ....\ \cos2^{n-1}\theta$

$$=\frac{2\sin\theta\cos\theta\cdot\cos2\theta\cos2^2\theta.....\cos2^{n-1}\theta}{2\sin\theta}$$

$$=\frac{2\sin2\theta\cos2\theta\cos^2 2\theta.....\cos2^{n-1}\theta}{2^2\sin\theta}$$

$$\text{--}$$

$$\text{--}$$

$$=\frac{\sin2^n\theta}{2^n\sin\theta}=\frac{\sin(\pi-\theta)}{2^n\sin\theta}=\frac{1}{2^n}\qquad ...\ (2)$$

⇒ Using A.M. ≥ G.M., we have

$$\frac{1+2\cos\theta}{2}\ge\sqrt{1\cdot2\cos\theta}$$

$$\frac{1+2\cos2\theta}{2}\ge\sqrt{1\cdot2\cos2\theta}$$

$$\text{-----------------------------------}$$

$$\text{-----------------------------------}$$

$$\frac{1+2\cos2^{n-1}\theta}{2}\ge\sqrt{1\cdot2\cos2^{n-1}\theta}$$

On multiplying,

$$\left(\frac{1+2\cos\theta}{2}\right)\left(\frac{1+2\cos2\theta}{2}\right)....\left(\frac{1+2\cos2^{n-1}\theta}{2}\right)\ge$$

$$\sqrt{2^n\cos\theta\cos2\theta\cos2^2\theta.....\cos2^{n-1}\theta}$$

$$\Rightarrow\ \frac{1}{2^n}\prod_{r=0}^{n-1}(1+2\cos2^r\theta)\ge1\qquad\text{[From (2)]}$$

$$\therefore\ \prod_{r=0}^{n-1}(1+2\cos2^r\theta)\ge2^n$$

6. (b) : In ΔABC, $a + b > c$ and $a > 0$

$$\Rightarrow\ \frac{a+b-c}{a}>0\ \Rightarrow\ \left(1+\frac{b-c}{a}\right)>0\ \text{etc.}$$

∴ Using A.M. ≥ G.M. for weighted means, we get

$$\frac{a\left(1+\frac{b-c}{a}\right)+b\left(1+\frac{c-a}{b}\right)+c\left(1+\frac{a-b}{c}\right)}{a+b+c}$$

$$\ge\sqrt[a+b+c]{\left(1+\frac{b-c}{a}\right)^a\left(1+\frac{c-a}{b}\right)^b\left(1+\frac{a-b}{c}\right)^c}$$

$$\Rightarrow\left\{\frac{(a+b-c)+(b+c-a)+(c+a-b)}{a+b+c}\right\}^{\frac{1}{a+b+c}}$$

$$\ge\left(1+\frac{b-c}{a}\right)^a\left(1+\frac{c-a}{b}\right)^b\left(1+\frac{a-b}{c}\right)^c$$

$$\Rightarrow\left(1+\frac{b-c}{a}\right)^a\left(1+\frac{c-a}{b}\right)^b\left(1+\frac{a-b}{c}\right)^c\le1$$

$$\Rightarrow\prod\left(1+\frac{b-c}{a}\right)^a\le1$$

Clearly,

$$\prod\left(1+\frac{b-c}{a}\right)^a>0\ \therefore\ 0<\prod\left(1+\frac{b-c}{a}\right)^a\le1$$

for $n > 1$.

7. (b) : $\because\ \alpha+\beta=\frac{5\pi}{4}$ ... (1)

$$\therefore\ \cot(\alpha+\beta)=\cot\frac{5\pi}{4}\ \Rightarrow\ \frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}=1$$

⇒ $\cot\alpha + \cot\beta + 1 = \cot\alpha\cot\beta$

⇒ $(1 + \cot\alpha)(1 + \cot\beta) = 2\cot\alpha\cot\beta$

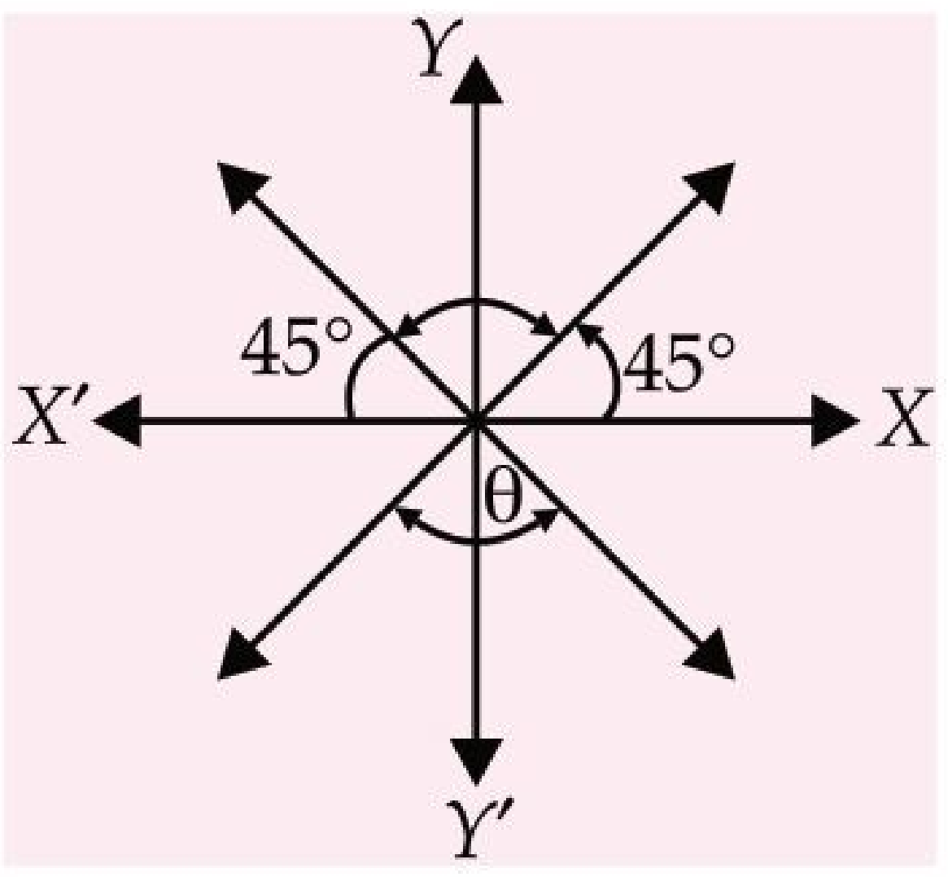
$$\Rightarrow\ \frac{\cot\alpha}{1+\cot\alpha}\cdot\frac{\cot\beta}{1+\cot\beta}=\frac{1}{2}\qquad ...\ (2)$$

Now, $f(\theta) = \cot\theta - \cot^2\theta + \cot^3\theta -$ ..... to ∞

$$=\frac{\cot\theta}{1+\cot\theta},\ \text{provided}\ -1<\cot\theta<1\qquad ...\ (3)$$

$\left[\because\right.$ For an infinite G.P., Sum $=\frac{a}{1-r}$, where common ratio $\left.\in(-1,1)\right]$

$$\therefore\ \text{(3) is satisfied if}\ \theta\in\left(n\pi+\frac{\pi}{4},\,n\pi+\frac{3\pi}{4}\right)\qquad ...(4)$$



∴ $f(\alpha), f(\beta)$ must satisfy (4) also simultaneously (1)

(a) From (2), we can say $f(\alpha)f(\beta)=\frac{1}{2}$, but subject to (4).

$\therefore f(\alpha)f(\beta)=\frac{1}{2}$ is not correct always.

(b) $\because \frac{2\pi}{3}+\frac{7\pi}{12}=\frac{5\pi}{4}$ and $\frac{2\pi}{3},\frac{7\pi}{12}$ both satisfy (4)

$$\therefore f\left(\frac{2\pi}{3}\right)f\left(\frac{7\pi}{12}\right)=\frac{1}{2}=f\left(\frac{\pi}{4}\right)$$

(c) $\because \frac{\pi}{3}+\frac{11\pi}{12}=\frac{5\pi}{4}$ but $\frac{11\pi}{12}$ does not satisfy (4)

∴ (c) is incorrect.

(d) $\because \frac{5\pi}{12}+\frac{5\pi}{6}=\frac{5\pi}{4}$ but $\frac{5\pi}{6}$ does not satisfy (4)

∴ (d) is incorrect.

8. (d) : $a_1=\lim_{x\to 0^+}\{1+(\sin x)^{1/x}\}=1+0=1$

$$\left[\begin{array}{l}\because 0<\sin x<1 \text{ as } x\to 0^+\\ \text{Also, } \frac{1}{x}\to\infty\Rightarrow(\sin x)^{1/x}\to 0\end{array}\right]$$

$\because a_{n-1}(a_n-a_{n-1})=1 \therefore a_n=a_{n-1}+\frac{1}{a_{n-1}}$

$\Rightarrow a_n^2=a_{n-1}^2+2+\frac{1}{a_{n-1}^2}$ ...(1)

$$\therefore \begin{array}{l} a_n^2>a_{n-1}^2+2\\ a_2^2>a_1^2+2\\ a_3^2>a_2^2+2\\ \ldots\ldots\ldots\ldots\\ \ldots\ldots\ldots\ldots\\ a_{403}^2>a_{402}^2+2\end{array}$$

On adding, we get $a_{403}^2>a_1^2+2\times 402$

i.e., $a_{403}^2>805$ ...(2)

Now, $a_2=a_1+\frac{1}{a_1}=2$ $[\because a_1=1]$

$\therefore$ For $n>1$, $a_n>1$ $\therefore \frac{1}{a_n^2}<1$

$\therefore$ From (1), $a_n^2<a_{n-1}^2+3$ $(n>2)$

$$\therefore \left.\begin{array}{l} a_3^2<a_2^2+3\\ a_4^2<a_3^2+3\\ \ldots\ldots\ldots\ldots\\ a_{403}^2<a_{402}^2+3\end{array}\right\}$$

On adding, we get $a_{403}^2<a_2^2+3\times 401$

$\Rightarrow a_{403}^2<1207$ ...(3)

From (2) and (3) we have $a_{403}^2\in(805,1207)$

$\Rightarrow p=805,\ q=1207 \therefore p+q=2012.$

9. (a) : Given hyperbola is $\frac{y^2}{16}-\frac{x^2}{25}=1$ ... (i)

$\therefore$ (h, k) is a point on it $\therefore \frac{k^2}{16}-\frac{h^2}{25}=1$

$\Rightarrow 25k^2-16h^2=400$ ... (ii)

$\therefore$ Tangent at (h, k) is $\frac{ky}{16}-\frac{hx}{25}=1$

$\Rightarrow y=\frac{16hx}{25k}+\frac{16}{k}$ ... (iii)

Given circle is $x^2+y^2=16$ ... (iv)

On solving (iii) and (iv), we get

$$x^2+\left(\frac{16hx}{25k}+\frac{16}{k}\right)^2=16$$

$$\Rightarrow \left(1+\frac{256h^2}{625k^2}\right)x^2+\frac{512hx}{25k^2}+\left(\frac{256}{k^2}-16\right)=0$$

According to problem, its two roots are x_1 and x_2

$$\therefore \frac{x_1+x_2}{x_1x_2}=\frac{\frac{-512h}{25k^2}}{\left(\frac{256}{k^2}-16\right)} \quad \left[\because \frac{(-b/a)}{c/a}=-\frac{b}{c}\right]$$

$$=\frac{512h}{25(16k^2-256)}=\frac{32h}{25k^2-400}=\frac{32h}{16h^2} \quad \text{[from (ii)]}$$

$\frac{1}{x_1}+\frac{1}{x_2}=\frac{2}{h} \Rightarrow x_1,h,x_2$ are in H.P.

10. (a) : $(x+a)^{100}=C_0x^{100}+C_1x^{99}\cdot a+\ldots+C_{100}a^{100}$

$=(C_0x^{100}+C_2x^{98}a^2+\ldots+C_{100}a^{100})$

$+(C_1x^{99}a+C_3x^{97}a^3+\ldots.+C_{99}xa^{99})$

$=(t_0+t_2+\ldots+t_{100})+(t_1+t_3+\ldots+t_{99})$ ...(i)

On replacing a by ia and $(-ia)$ respectively, we get

$(x+ia)^{100}=(t_0+t_2.i^2+t_4.i^4+\ldots+t_{100}.i^{100})$

$+(t_1\cdot i+t_3.i^3+\ldots+t_{99}\cdot i^{99})$

$=(t_0-t_2+t_4-\ldots+t_{100})+i(t_1-t_3+t_5-\ldots-t_{99})$ ... (ii)

And $(x-ia)^{100}=(t_0-t_2+t_4-\ldots+t_{100})$

$-i(t_1-t_3+t_5-\ldots-t_{99})$ ... (iii)

On multiplying (ii) and (iii), we get

$(x^2+a^2)^{100}=(t_0-t_2+t_4-\ldots+t_{100})^2$

$+(t_1-t_3+t_5-\ldots-t_{99})^2$ ... (iv)

$$\therefore \int \frac{2x\,dx}{\left(\sum_{r=0}^{50}(-1)^r t_{2r}\right)^2+\left(\sum_{r=0}^{49}(-1)^r t_{2r+1}\right)^2}$$

$$= \int \frac{2x\,dx}{\left(t_0-t_2+t_4-...+t_{100}\right)^2+\left(t_1-t_3+t_5-...-t_{99}\right)^2}$$

$$= \int \frac{2x\,dx}{\left(x^2+a^2\right)^{100}} \text{ put } x^2+a^2=z \;\Rightarrow 2x\,dx=dz$$

$$= \int \frac{dz}{z^{100}} = \frac{z^{-100+1}}{-100+1}+C$$

$$= -\frac{1}{99z^{99}}+C = \frac{-1}{99(x^2+a^2)^{99}}+C$$

11. (d): $11^{2012} + 23^{2012} - 3^{2012}$

$= (1 + 10)^{2012} + (3 + 20)^{2012} - 3^{2012}$

$$=\left\{1+{}^{2012}C_1\cdot 10+{}^{2012}C_2\cdot 10^2+.....\right\}$$
$$+\left\{3^{2012}+{}^{2012}C_1\cdot 3^{2011}\cdot 20+.....\right\}-3^{2012}$$

$$=\left\{{}^{2012}C_1\cdot 10+{}^{2012}C_2\cdot 10^2+.....\right\}$$
$$+\left\{{}^{2012}C_1\cdot 3^{2011}20+{}^{2012}C_2\cdot 3^{2010}\cdot 20^2+.....\right\}+1$$

= (All terms multiple of 10) + 1

$\therefore$ Unit place digit = 1 $\Rightarrow a = 1$

$$\text{Let } I=\int_{a-1}^{a}\frac{dx}{\sqrt{1-x^2}-x+\frac{1}{x}}=\int_0^1\frac{x\,dx}{x\sqrt{1-x^2}+1-x^2}$$

$$=\int_0^1\frac{x\,dx}{\left(x+\sqrt{1-x^2}\right)\sqrt{1-x^2}}$$

Putting $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

x	0	1
θ	0	$\pi/2$

$$=\int_0^{\pi/2}\frac{\sin\theta\cos\theta\,d\theta}{(\sin\theta+\cos\theta)\cos\theta}=\int_0^{\pi/2}\frac{\sin\theta\,d\theta}{\sin\theta+\cos\theta} \quad ....(1)$$

Again

$$I=\int_0^{\pi/2}\frac{\sin\theta\;d\theta}{\sin\theta+\cos\theta}=\int_0^{\pi/2}\frac{\sin\left(\frac{\pi}{2}-\theta\right)d\theta}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)}$$

$$=\int_0^{\pi/2}\frac{\cos\theta\;d\theta}{\cos\theta+\sin\theta} \quad ...(2)$$

On adding (1) & (2), $2I=\int_0^{\pi/2}d\theta \;\;\therefore\;\; I=\frac{\pi}{4}$

12. (d): On integrating, we get

$$\frac{ax^2}{2}+hx=\frac{by^2}{2}+ky+c$$

$\Rightarrow\; ax^2 - by^2 + 2hx - 2ky - 2c = 0$ … (1)

$\because$ General 2nd degree equation

$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + C = 0$

represents a parabola, if $H^2 = AB$

$\therefore$ we must have, $0^2 = a(-b)$

[$\because$ Here $A = a$, $H = 0$, $B = -b$]

$\therefore$ $ab = 0$ … (2)

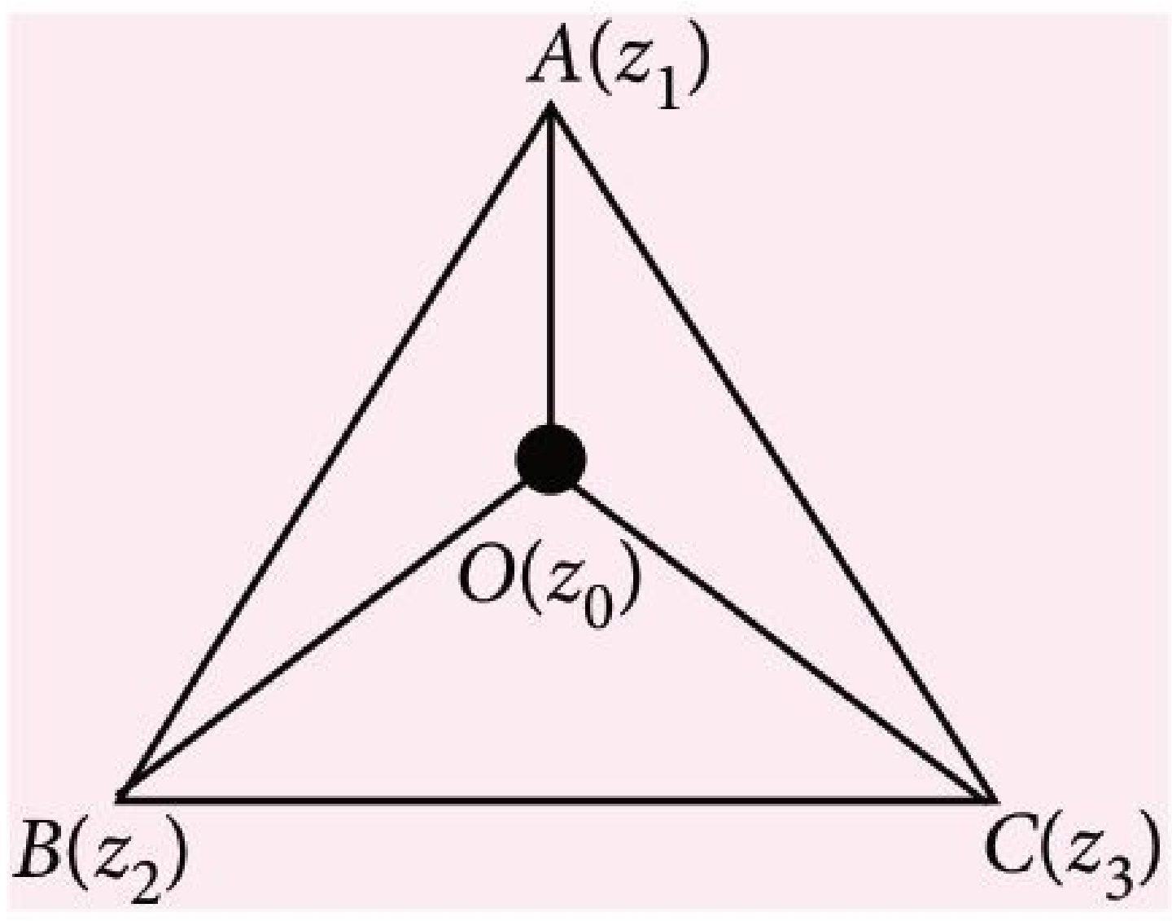
$\because$ a and b both can't be zero simultaneously to get parabola from (1)

To satisfy (2) also, only one of a and b should be zero.

13. (c) : Taking rotation at O

$$\frac{z_0-z_1}{z_0-z_2}=\cos 2C-i\sin 2C$$

$$\frac{z_0-z_3}{z_0-z_2}=\cos 2A+i\sin 2A$$



$$\text{Now } \left(\frac{z_0-z_1}{z_0-z_2}\right)\frac{\sin 2A}{\sin 2B}+\left(\frac{z_0-z_3}{z_0-z_2}\right)\frac{\sin 2C}{\sin 2B}$$

$$=\frac{\sin 2A\cos 2C-i\sin 2A\sin 2C+\cos 2A\sin 2C+i\sin 2A\sin 2C}{\sin 2B}$$

$$=\frac{\sin(2A+2C)}{\sin 2B}=-1$$

14. (b): $I=\left|(\vec{b}-\vec{a})\cdot\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| \Rightarrow |\vec{b}-\vec{a}||\vec{p}\times\vec{q}|\cos 60^\circ=\frac{1}{2}$

$$\Rightarrow AB\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{2} \;\Rightarrow\; AB=2$$

15. (a, d) : If $x\in\left(\frac{5\pi}{2},3\pi\right)$, $\sin^{-1}(\sin x)=3\pi-x$,

$$\cos^{-1}(\cos x)=x-2\pi,\tan^{-1}(\tan x)=x-3\pi$$

◆◆

beat the TIME TRAP

Duration : 60 minutes

SECTION-I

Only One Option Correct Type

1. The function $y = x - \cot^{-1} x - \log\left|x - \sqrt{x^2+1}\right|$ is increasing on

(a) (0, 0) (b) $(-\infty, 0)$ (c) $(0, \infty)$ (d) $(-\infty, \infty)$

2. An infinite G. P. has 2nd term x and its sum is 4, then x belongs to

(a) (0, 2] (b) (1, 8)

(c) (−8, 1] (d) none of these

3. $\sum_{r=0}^{10} 2^r \, {}^{20}C_r \, {}^{20-r}C_{10-r}$ is equal to

(a) ${}^{20}C_{10}$ (b) ${}^{20}C_{10}\left(\frac{3}{2}\right)^{10}$

(c) ${}^{20}C_{10}\, 3^{10}$ (d) ${}^{20}C_{10}\, 2^{10}$

4. If the graphs of the functions $y = \ln x$ and $y = ax$ intersect at exactly two points, then a must be

(a) $(0, e)$ (b) $(1/e, 0)$

(c) $(0, 1/e)$ (d) none of these

5. If $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} = \frac{\pi}{2ab}$, then the value of $\int_0^{\pi/2} \frac{dx}{(4\cos^2 x + 9\sin^2 x)^2}$ is equal to

(a) $\frac{11\pi}{864}$ (b) $\frac{13\pi}{864}$

(c) $\frac{17\pi}{864}$ (d) none of these

6. If $(3 + x^{2008} + x^{2009})^{2010} = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$, then the value of $a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 - ...$ is

(a) 3^{2010} (b) 1

(c) 2^{2010} (d) none of these

7. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ which subtends an angle of 90° at the point (2, 3), then the locus of the mid point of AB is circle whose centre is

(a) (1, 5) (b) (1, 5/2)

(c) (1, 3/2) (d) (2, 5/2)

8. If k be a positive integer, then the number of values of k satisfying

$$\int_0^{\pi/2}\left[k^2\left(\cos 3x + \frac{1}{2}\cos x\right) + \sin x - 2k\cos x\right]dx \le 1,$$ is

(a) 10 (b) 11

(c) 12 (d) none of these

9. The solution of differential equation
$(x\cot y + \ln\cos x)dy + (\ln\sin y - y\tan x)dx = 0$ is

(a) $(\sin x)^y (\cos y)^x = c$ (b) $(\sin y)^x (\cos x)^y = c$

(c) $(\sin x)^y (\sin y)^x = c$ (d) $(\cot x)^y (\cot y)^x = c$

10. The value of $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.\,{}^nC_r\,{}^nC_{r+1}}{r+2}\right)$ is

(a) ${}^{2n-1}C_{n+1}$ (b) ${}^{2n}C_{n-1}$

(c) ${}^{2n-1}C_{n-1}$ (d) ${}^{2n}C_{n-2}$

11. The number of points in the cartesian plane with integral co-ordinates satisfying the inequations $|x| \le 10$, $|y| \le 10$ and $|x - y| \le 10$ is

(a) 321 (b) 331

(c) 341 (d) none of these

12. If $f(x + y) = f(x) - f(y) + 2xy - 1\ \forall\ x, y \in R$. Also if $f(x)$ is differentiable and $f'(0) = b$ also $f(x) > 0\ \forall\ x$, then the set of values of b is

(a) ϕ (b) $\{1\}$
(c) $\{1, 2\}$ (d) none of these

13. Let $a_1 = \left(\tan\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$, $a_2 = \left(\tan\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$, $a_3 = \left(\cot\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$, $a_4 = \left(\cot\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$, then which of the following statement(s) is/are true?

(a) $a_4 > a_3 > a_2 > a_1$ (b) $a_3 > a_4 > a_2 > a_1$
(c) $a_3 > a_4 > a_1 > a_2$ (d) $a_3 > a_1 > a_2 > a_4$

14. The equation of curve passing through (1, 1) in which the subtangent is always bisected at the origin, is

(a) $y^2 = x$ (b) $2x^2 - y^2 = 1$
(c) $x^2 + y^2 = 2$ (d) $x + y = 2$

15. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors such that $\vec{b} \times \vec{c} = \vec{a}$, $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{c} \times \vec{a} = \vec{b}$ then

(a) $[\vec{a}\ \vec{b}\ \vec{c}] = 1$ (b) $[\vec{a}\ \vec{b}\ \vec{c}] \neq 1$
(c) $|\vec{a}| + |\vec{b}| + |\vec{c}| = 5$ (d) none of these

16. If in a ΔABC, $\cos A + 2\cos B + \cos C = 2$, then a, b, c are in

(a) A.P. (b) H.P. (c) G.P. (d) A.G.P.

17. The value of $a (a < 0)$ for which least value of quadratic expression $4x^2 - 4ax + a^2 - 2a + 2$ on the interval $0 \le x \le 2$ is equal to 3, is

(a) $-\sqrt{2}$ (b) $-\sqrt{2} - 2$
(c) $1 - \sqrt{2}$ (d) none of these

18. If $f(x) = \tan\frac{3\pi}{2}[x] + \cos^2 \pi[x] + e^{\left\{\frac{x}{3}\right\}}$ (where [.] denotes the greatest integer function and {.} denotes fractional part), then

(a) $f(x)$ is periodic with fundamental period not defined
(b) $f(x)$ is periodic with period 6
(c) $f(x)$ is non periodic
(d) $f(x)$ is periodic with period 2

19. If the shortest distance between lines
$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda_1(2\hat{i} + 3\hat{j} + 4\hat{k})$ and
$\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \lambda_2(3\hat{i} + 4\hat{j} + 5\hat{k})$ is x.

Then, $\cos^{-1} \cos \sqrt{6}\, x$ is equal to

(a) 1/2 (b) 0 (c) 1 (d) π

20. The maximum values of $\frac{\log x}{x}$ is

(a) 1 (b) $2/e$ (c) e (d) $1/e$

21. The maximum value of $\cos(\sin x) + \sin(\cos x)$, $0 \le x \le \pi$, is

(a) $1 + \sin 1$ (b) $\cos 1$
(c) $1 + \cos 1$ (d) $2\sin^2 \frac{1}{2}$

22. The sides of the triangle are in the ratio $1 : \sqrt{3} : 2$, then the corresponding angles of triangle are in the ratio

(a) 1 : 3 : 4 (b) 2 : 3 : 4
(c) 3 : 2 : 1 (d) 1 : 2 : 3

SECTION-II

One or More than One Option(s) Correct Type

23. Tangents are drawn from a point P on the circle $C : x^2 + y^2 = a^2$ to the circle $C_1 : x^2 + y^2 = b^2$. If these tangents cut the circle C at Q and R and if QR is a tangent to the circle C_1, then the area of triangle PQR is

(a) $3\sqrt{3}\, b^2$ (b) $\frac{3\sqrt{3}}{4} a^2$
(c) $\frac{3\sqrt{3}}{2} ab$ (d) $2\sqrt{3}\, ab$

24. Triangles $A_1'A_2'A_3'$ and $B_1'B_2'B_3'$ have side lengths (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively satisfying the relation

$$\sqrt{a_1 + a_2 + a_3}\sqrt{b_1 + b_2 + b_3} = \sqrt{a_1 b_1} + \sqrt{a_2 b_2} + \sqrt{a_3 b_3},$$

then which one of the following statements is/are true ?

(a) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$
(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = 1$
(c) $\Delta A_1'A_2'A_3'$ and $\Delta B_1'B_2'B_3'$ are similar
(d) $\Delta A_1'A_2'A_3'$ and $\Delta B_1'B_2'B_3'$ are congruent

25. If the equation $\bar{a}z + a\bar{z} + b = 0$ and $\bar{a}z - a\bar{z} + b_1 = 0$ represent two lines L_1 and L_2 in the complex plane then

(a) L_1 and L_2 are perpendicular
(b) b is purely real
(c) b_1 is purely imaginary
(d) b_1 is purely real

26. Let $f: R \to R$ be a function defined by $f(x+1) = \dfrac{f(x)-5}{f(x)-3}\ \forall \in R$. Then which of the following statement(s) is/are true?

(a) $f(2008) = f(2004)$ (b) $f(2006) = f(2010)$
(c) $f(2006) = f(2002)$ (d) $f(2006) = f(2018)$

27. If the equation $\sec x + \operatorname{cosec} x = k$, $(k \in R)$ has exactly 3 solutions between 0 and π, then k can take values

(a) $\sqrt{2}$ (b) 2 (c) $3\sqrt{2}$ (d) 4

SECTION-III

Comprehension Type

Let $\vec{r}$ is a position vector of a variable point in cartesian OXY plane such that $\vec{r} \cdot (10\hat{j} - 8\hat{i} - \vec{r}) = 40$ and

$p_1 = \max\{|\vec{r} + 2\hat{i} - 3\hat{j}|^2\}$, $p_2 = \min\{|\vec{r} + 2\hat{i} - 3\hat{j}|^2\}$.

A tangent line is drawn to the curve $y = \dfrac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B.

28. p_2 is equal to

(a) 9 (b) $2\sqrt{2}-1$ (c) $6\sqrt{2}+3$ (d) $9-4\sqrt{2}$

29. $p_1 + p_2$ is equal to

(a) 2 (b) 10 (c) 18 (d) 5

30. $\overrightarrow{AB} \cdot \overrightarrow{OB}$ is

(a) 1 (b) 2 (c) 3 (d) 4

SECTION-IV

Matrix-Match Type

31. Match the following :

	Column-I		Column-II
(P)	If $y = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(b)$, $(0 < b < 1)$ and $0 < y \le \pi/4$, then the maximum value of b will be	(1)	2
(Q)	The number of solutions of $\sin^4 x + \cos^3 x \ge 1$ in $(0, 2\pi)$ will be	(2)	1/3
(R)	The area bounded by the loop of $4y^2 = x^2(4 - x^2)$ is	(3)	1/2
(S)	If $(1+x^2)\frac{dy}{dx} + 2xy - 1 = 0$ passes through origin, then $y(1)$ will be	(4)	16/3

32. Match the following :

	Column-I		Column-II
(P)	The differential equation $y = x\frac{dy}{dx} + \frac{dx}{dy}$ represents	(1)	pair of straight lines
(Q)	If an ellipse slides between two perpendicular straight lines, then the locus of its centre is	(2)	straight line
(R)	If a, b are real numbers and $c > 0$, then the locus of (x,y) represented by the equation $\lvert ay - bx \rvert = c\sqrt{(x-a)^2 + (y-b)^2}$ is	(3)	circle
(S)	If $h \ne 0$ and $\frac{a}{h} = \frac{\sqrt{2}c}{b}$, then the equation $hxy + ax + by + c = 0$ represents	(4)	parabola
		(5)	hyperbola

33. Match the following :

	Column-I		Column-II
(P)	$\vec{a}, \vec{b}$ are unit vectors and $\vec{a} + 2\vec{b} \perp 5\vec{a} - 4\vec{b}$, then $2(\vec{a} \cdot \vec{b})$ is equal to	(1)	0
(Q)	The points (1, 0, 3), (–1, 3, 4), (1, 2, 1), (k, 2, 5) are coplanar, then k =	(2)	–1
(R)	The vectors (1, 1, m), (1, 1, m + 1), (1, –1, m) are coplanar, then number of values of m is	(3)	1
(S)	$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$ is equal to	(4)	2

34. Match the following :

	Column-I		Column-II
(P)	If the line $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z+1}{\lambda}$ lies in the plane $3x - 2y + 5z = 0$, then λ is equal to	(1)	$\sin^{-1}\sqrt{\frac{6}{25}}$
(Q)	If $(3, \lambda, \mu)$ is a point on the line $2x + y + z - 3 = 0 = x - 2y + z - 1$, then $\lambda + \mu$ is equal to	(2)	–7/5

(R)	The angle between the line $x = y = z$ and the plane $4x - 3y + 5z = 2$ is	(3)	-3
(S)	The angle between the planes $x + y + z = 0$ and $3x - 4y + 5z = 0$ is	(4)	$\cos^{-1}\sqrt{\frac{8}{75}}$

35. Match the following :

	Column-I		Column-II
(P)	The number of arrangements of the word BANANA in which two N's do not appear together is	(1)	$[1, 2)$
(Q)	If $(1+x)^n = C_0 + C_1x + ... + C_nx^n$, then $\left(1+\frac{C_1}{C_0}\right)...\left(1+\frac{C_n}{C_{n-1}}\right)$ is	(2)	$\frac{(n+1)^n}{n!}$
(R)	If c, d, e are in A.P., then $\Delta = \begin{vmatrix} x+1 & x+2 & x+c \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}$ is	(3)	$\frac{2(n+1)^n}{n!}$
(S)	$\frac{c^2+d^2+e^2}{cd+de+ce}$ lies between (where c, d, e are the lengths of the sides of ΔCDE)	(4)	0
		(5)	40

SECTION-V

Integer Answer Type

36. If $f(x + y + z) = f(x) + f(y) + f(z)$ with $f(1) = 1$ and $f(2) = 2$ and $x, y, z \in N$, then $\lim\limits_{n\to\infty} \frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3}$ is equal to ________.

37. $x, y \in R$, $x^2 + y^2 + xy = 1$, then the minimum value of $x^3y + xy^3 + 4$ is _____.

38. If the equation of the curve on reflection of the ellipse $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$ about the line $x - y - 2 = 0$ is $16x^2 + 9y^2 + k_1x - 36y + k_2 = 0$, then $\frac{k_1+k_2}{44}$ is ______.

39. If the number of ordered pairs of (x, y) satisfying the system of equations $5x\left(1+\frac{1}{x^2+y^2}\right) = 12$ and $5y\left(1-\frac{1}{x^2+y^2}\right) = 4$ is n, then n is ________.

40. If x, y, z are distinct positive numbers such that $x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$, then the value of xyz is ______.

SOLUTIONS

1. (d) : $y = x - \cot^{-1}x - \log\left|x - \sqrt{x^2+1}\right|$,

$$\frac{dy}{dx} = 1 + \frac{1}{1+x^2} + \frac{1}{\sqrt{x^2+1}} \text{ as } \frac{dy}{dx} > 0 \text{ for all } R,$$

So, y is increasing on $(-\infty, \infty)$.

2. (c) : Sum$=\frac{\frac{x}{r}}{1-r} = 4$ (where r is common ratio)

$x = 4r(1 - r) = 4(r - r^2)$

For $r \in (-1, 1)$, $r - r^2 \in \left(-2, \frac{1}{4}\right] \Rightarrow x \in (-8, 1]$

3. (c) : $\sum_{r=0}^{10} 2^r \, {}^{20}C_r \, {}^{20-r}C_{10-r}$

$$= \sum_{r=0}^{10} 2^r \frac{20!}{r!(20-r)!} \times \frac{(20-r)!}{10!\,(10-r)!}$$

$$= \sum_{r=0}^{10} 2^r \frac{20!}{10!\,10!} \times \frac{10!}{r!(10-r)!} = \sum_{r=0}^{10} 2^r \, {}^{20}C_{10} \, {}^{10}C_r = {}^{20}C_{10}\, 3^{10}$$

4. (c) : Given curves are $y = \ln x$ and $y = ax$

$\Rightarrow$ $\ln x = ax$ has exactly two solutions.

$\Rightarrow$ $\frac{\ln x}{x} = a$ has exactly two solutions.

Let $y = \frac{\ln x}{x}$, $x > 0$, $\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1-\ln x}{x^2}$

y is increasing if $1 - \ln x > 0$, $\ln x < 1$, $0 < x < e$, range of y is $\left(-\infty, \frac{1}{e}\right]$. Thus, for exactly two solutions of $\frac{\ln x}{x} = a$, $a \in \left(0, \frac{1}{e}\right)$.

5. (b) : Let $I = \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{2ab}$

Differentiating w.r.t. a, we get

$$\int_0^{\pi/2} \frac{-dx 2a\cos^2 x}{(a^2\cos^2 x+b^2\sin^2 x)^2} = -\frac{\pi}{2a^2 b}$$

$$\Rightarrow \int_0^{\pi/2} \frac{\cos^2 x dx}{(a^2\cos^2 x+b^2\sin^2 x)^2} = \frac{\pi}{4a^3 b} \qquad \text{...(i)}$$

Similarly, differentiating w.r.t. b, we get

$$\int_0^{\pi/2} \frac{\sin^2 x dx}{(a^2\cos^2 x+b^2\sin^2 x)^2} = \frac{\pi}{4ab^3} \qquad \text{...(ii)}$$

Adding (i) & (ii), we get

$$\int_0^{\pi/2} \frac{dx}{(a^2\cos^2 x+b^2\sin^2 x)^2} = \frac{\pi}{4ab}\left[\frac{1}{a^2}+\frac{1}{b^2}\right]$$

$$\therefore \int_0^{\pi/2} \frac{dx}{(4\cos^2 x+9\sin^2 x)^2} = \frac{\pi}{24}\left[\frac{1}{4}+\frac{1}{9}\right] = \frac{13\pi}{864}$$

6. (c) : Putting $x = \omega, \omega^2$, we get

$(3 + \omega + \omega^2)^{2010} = a_0 + a_1\omega + a_2\omega^2 + ...$

$$\Rightarrow 2^{2010} = a_0 + a_1\omega + a_2\omega^2 + a_3 + a_4\omega + ... \qquad \text{...(i)}$$

$$\text{and } 2^{2010} = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega^2 + ... \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2^{2011} = 2a_0 - a_1 - a_2 + 2a_3 - a_4 - a_5 + 2a_6 - ...$$

$$2^{2010} = a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 - ....$$

7. (d) : Let mid point of AB is $M(h, k)$.

AB subtends 90° at (2, 3).

$$\Rightarrow AM = MB = \sqrt{(h-2)^2+(k-3)^2}$$

Also, $CM^2 + MB^2 = CB^2$

$\Rightarrow (h - 2)^2 + (k - 2)^2 + (h - 2)^2 + (k - 3)^2 = 4$

$$\Rightarrow x^2 + y^2 - 4x - 5y + \frac{17}{2} = 0$$

8. (c) : $\left[k^2\left(\frac{\sin 3x}{3}+\frac{1}{2}\sin x\right) - \cos x - 2k\sin x\right]_0^{\pi/2} \le 1$

$$\Rightarrow \left\{k^2\left(\frac{-1}{3}+\frac{1}{2}\right) - 0 - 2k\right\} - \{-1\} \le 1 \Rightarrow \frac{k^2}{6} - 2k + 1 \le 1$$

$\Rightarrow k(k - 12) \le 0, 0 \le k \le 12, k = 1, 2, 3, 4, ...,12.$

9. (b) : $(x\cot y + \ln\cos x)dy + (\ln\sin y - y\tan x)dx = 0$

$\Rightarrow x\cot y\, dy + \ln\sin y\, dx + \ln\cos x\, dy - y\tan x\, dx = 0$

$\Rightarrow \int d(x\ln\sin y) + \int d(y\ln\cos x) = 0$

$\Rightarrow x \ln\sin y + y \ln\cos x = \ln k \Rightarrow (\sin y)^x(\cos x)^y = c$

10. (d) : $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.{}^nC_r\,{}^nC_{r+1}}{r+2}\right) = \sum_{r=0}^{n-1} {}^{n-1}C_{r-1}\,{}^{n+1}C_{r+2}$

$$= {}^{n-1}C_0\,{}^{n+1}C_3 + ... + {}^{n-1}C_{n-2}\,{}^{n+1}C_{n+1}$$

$$(1+x)^{n-1} = {}^{n-1}C_0 + {}^{n-1}C_1x + {}^{n-1}C_2x^2 + ... + {}^{n-1}C_{n-1}x^{n-1}$$

$$(1+x)^{n+1} = {}^{n+1}C_0x^{n+1} + {}^{n+1}C_1x^n + {}^{n+1}C_2x^{n-1} + ... + {}^{n+1}C_{n+1}$$

Coefficient of x^{n-2} in $(1+x)^{2n} = {}^{2n}C_{n-2}$.

11. (b) : $|x| \le 10 \Rightarrow -10 \le x \le 10,$
$|y| \le 10 \Rightarrow -10 \le y \le 10,$
$|x - y| \le 10 \Rightarrow -10 \le x - y \le 10$

Total number of points
$= (20 + 1)^2 - 2[1 + 2 + ... + 10] = 331$

12. (a) : $f(0) = f(0) - f(0) + 2(0) - 1 = -1$

$$\text{Now, } f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x)-f(h)+2xh-1-f(x)}{h}$$

$$= \left\{\lim_{h\to 0}\frac{-f(h)+f(0)}{h}\right\} + 2x \Rightarrow f'(x) = 2x - f'(0) = 2x - b$$

$\therefore f(x) = x^2 - bx + c$

But $f(0) = -1$

$\Rightarrow f(x) = x^2 - bx - 1, D = b^2 + 4 > 0\ \forall\ b \in R$

Hence, $f(x)$ cannot be > 0 always $\forall\ x \in R$.

13. (c) : As $0 < \frac{\pi}{8} < \frac{\pi}{4} \Rightarrow \cot\frac{\pi}{8} > 1 > \tan\frac{\pi}{8}$

$\Rightarrow a_3 > a_4$ and $a_1 > a_2$

Again, $\cot\frac{\pi}{8} > 1 > \tan\frac{\pi}{8} > 0 \Rightarrow a_1 < a_3$ and $a_1 < a_4$

Hence, $a_2 < a_1 < a_4 < a_3$.

14. (a) : TM is subtangent where $T \equiv \left(x - y\frac{dx}{dy}, 0\right)$

and $M \equiv (x, 0)$, $\frac{1}{2}\left(x - y\frac{dx}{dy} + x\right) = 0$

$$\Rightarrow 2x - y\frac{dx}{dy} = 0 \Rightarrow 2\int\frac{dy}{y} = \int\frac{dx}{x}$$

$\Rightarrow 2\ln y = \ln cx$

$\Rightarrow y^2 = cx$, curve passes through (1, 1).

$\therefore c = 1, y^2 = x$

15. (a) : $\vec{b}\times\vec{c} = \vec{a} \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot\vec{a} = |\vec{a}|^2$

Similarly, $\vec{a}\times\vec{b} = \vec{c} \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = |\vec{c}|^2$, And $\vec{c}\times\vec{a}=\vec{b} \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}]=|\vec{b}|^2$

Also, $[\vec{a}\,\vec{b}\,\vec{c}]=[\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}]=[\vec{a}\,\vec{b}\,\vec{c}]^2 \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}]=1$

16. (a) : $\cos A + \cos C = 2(1 - \cos B)$

$$\Rightarrow\ 2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=2\cdot 2\sin^2\frac{B}{2},$$

$$\Rightarrow\ 2\cos\frac{B}{2}\cos\frac{A-C}{2}=2\left(2\sin\frac{B}{2}\cos\frac{B}{2}\right)=2\sin B$$

$$\Rightarrow\ 2\sin\frac{A+C}{2}\cos\frac{A-C}{2}=2\sin B$$

$\Rightarrow$ $\sin A + \sin C = 2\sin B$, *i.e.* $a + c = 2b$.

17. (c) : Let $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$,

vertex of the parabola is $x=\frac{a}{2}$, as $a<0 \Rightarrow \frac{a}{2}<0$

For $f(x)$ to be minimum, $f'(x) = 0 \Rightarrow x=\frac{a}{2}<0$

So, for given values $f(x)$ will be minimum at $x = 0$.

$f(0)=a^2-2a+2=3 \Rightarrow a^2-2a-1=0$

$$\therefore\ a=\frac{2\pm\sqrt{4+4}}{2}=1\pm\sqrt{2},\ a=1-\sqrt{2}$$

18. (b) : $\tan\frac{3\pi}{2}[x+t] = \tan\frac{3\pi}{2}[x]$

$$\Rightarrow\ \frac{3\pi}{2}[x+t]=n\pi+\frac{3\pi}{2}[x] \Rightarrow 3([x+t]-[x])=2n$$

Clearly $n = 3$, $\therefore$ $t = 2$

$\cos^2\pi[x + t] = \cos^2\pi[x]$

$\Rightarrow$ $\pi[x + t] = n\pi + \pi[x]$; $[x + t] - [x] = n$

$\therefore$ $n = 1$

$\Rightarrow$ $t = 1$ and $e^{\{x/3\}}$ is periodic with period 3.

$\therefore$ LCM of 1, 2, 3 is 6.

19. (c) : $x=\left|\frac{(\hat{i}+2\hat{j}+2\hat{k}).(-\hat{i}+2\hat{j}-\hat{k})}{\sqrt{1+4+1}}\right|=\frac{1}{\sqrt{6}}$

Now, $\cos^{-1}\cos\sqrt{6}x=\cos^{-1}\cos 1=1$

20. (d) : $y=\frac{\log x}{x} \Rightarrow \frac{dy}{dx}=-\frac{1}{x^2}\log x+\frac{1}{x}\cdot\frac{1}{x}$

Now, $\frac{dy}{dx}=0 \Rightarrow \frac{1}{x^2}(1-\log x)=0$

$\Rightarrow$ $\log x = 1 \Rightarrow x = e$,

$x<e \Rightarrow \log x<1,\ x>e \Rightarrow \log x>1$

$\therefore$ At $x = e$, $\frac{dy}{dx}$ changes sign from positive to negative and hence, y has maximum value at $x = e$ and its value is $\frac{\log e}{e}=e^{-1}$.

21. (a) : $x = 0$ gives the maximum value.

22. (d) : Let the sides a, b, c be $\lambda, \sqrt{3}\lambda, 2\lambda$ respectively.

$c^2=a^2+b^2 \quad \therefore \quad \angle C=\frac{\pi}{2}$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{\lambda^2(3+4-1)}{\lambda^2 4\sqrt{3}}=\frac{\sqrt{3}}{2}$$

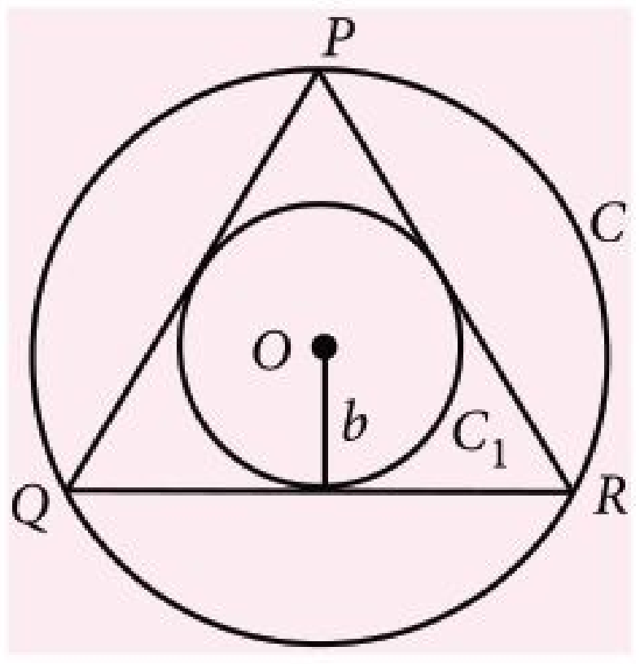
$$\therefore\ A=\frac{\pi}{6},\ B=\frac{\pi}{2}-A=\frac{\pi}{3}\quad\therefore\ A:B:C=1:2:3$$

23. (a, b, c) : All the sides of ΔPQR touch C_1.

$\therefore$ C_1 is incircle and C is the circumcircle with same centre.

The triangle is equilateral.

$\therefore$ $R = 2r \Rightarrow a = 2b$



Area of ΔPQR = $\frac{\sqrt{3}}{4}PR^2$

But $3b$ = $\frac{\sqrt{3}}{2}PR \Rightarrow PR=2\sqrt{3}b$

Area of ΔPQR = $\frac{\sqrt{3}}{4}\cdot 4\cdot 3b^2 = 3\sqrt{3}b^2$

$=\frac{3}{2}\sqrt{3}\,ab=\frac{3\sqrt{3}}{4}a^2$ [$\because$ $a = 2b$]

24. (a, c) : Since, $\sqrt{(a_1+a_2+a_3)(b_1+b_2+b_3)}$

$$=\sqrt{a_1b_1}+\sqrt{a_2b_2}+\sqrt{a_3b_3}$$

Let $a_1=A_1^2,\ a_2=A_2^2,\ a_3=A_3^2$

and $b_1=B_1^2,\ b_2=B_2^2,\ b_3=B_3^2$

$$\Rightarrow (A_1^2+A_2^2+A_3^2)(B_1^2+B_2^2+B_3^2)=(A_1B_1+A_2B_2+A_3B_3)^2$$

$$\Rightarrow\ (A_1B_2-A_2B_1)^2+(A_2B_3-A_3B_2)^2+(A_3B_1-A_1B_3)^2=0$$

$$\Rightarrow\ \frac{A_1}{B_1}=\frac{A_2}{B_2}=\frac{A_3}{B_3}=\lambda$$

or $\frac{\sqrt{a_1}}{\sqrt{b_1}}=\frac{\sqrt{a_2}}{\sqrt{b_2}}=\frac{\sqrt{a_3}}{\sqrt{b_3}} \quad \therefore\ \frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

25. (a, b, d)

26. (a, b, c, d) : $f(x+1)=\dfrac{f(x)-5}{f(x)-3}$ ....(i)

$f(x)\cdot f(x+1)-3f(x+1)=f(x)-5$

$\Rightarrow\ f(x)=\dfrac{3f(x+1)-5}{f(x+1)-1}$

Replacing x by $(x-1)$, we have

$$f(x-1)=\frac{3f(x)-5}{f(x)-1} \quad \text{...(ii)}$$

Using (i), we have

$$f(x+2)=\frac{f(x+1)-5}{f(x+1)-3}=\frac{\dfrac{f(x)-5}{f(x)-3}-5}{\dfrac{f(x)-5}{f(x)-3}-3}=\frac{2f(x)-5}{f(x)-2} \quad \text{...(iii)}$$

Using (ii), we have

$$f(x-2)=\frac{3f(x-1)-5}{f(x-1)-1}=\frac{3\left(\dfrac{3f(x)-5}{f(x)-1}\right)-5}{\dfrac{3f(x)-5}{f(x)-1}-1}=\frac{2f(x)-5}{f(x)-2}$$

...(iv)

Using (iii) and (iv), we have

$f(x+2)=f(x-2)\ \Rightarrow\ f(x+4)=f(x)$

$\Rightarrow$ $f(x)$ is periodic with period 4.

27. (c, d) : Let $f(x) = \sec x + \operatorname{cosec} x$ for $0 < x < \pi$, $x \neq \pi/2$.

$f'(x) = \sec x \tan x - \operatorname{cosec} x \cot x$

$= \sec^2 x \operatorname{cosec}^2 x(\sin^3 x - \cos^3 x)$

$\Rightarrow\ f'(x)<0$ for $0<x<\dfrac{\pi}{4}$ and

$f'(x)>0$ for $\dfrac{\pi}{4}<x<\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}<x<\pi$

$\Rightarrow$ $f(x)$ decreases strictly from ∞ to $f\left(\dfrac{\pi}{4}\right)$ for $0<x<\dfrac{\pi}{4}$

and increases from $f\left(\dfrac{\pi}{4}\right)$ to ∞ for $\dfrac{\pi}{4}<x<\dfrac{\pi}{2}$.

Also it increases strictly from $-\infty$ to ∞ for $\dfrac{\pi}{2}<x<\pi$.

Hence for any k the equation $f(x) = k$ has exactly one solution in $\left(\dfrac{\pi}{2},\ \pi\right)$.

If $f\left(\dfrac{\pi}{4}\right)<k<\infty$, then the equation $f(x) = k$ has exactly one solution in each of the intervals $\left(0,\ \dfrac{\pi}{4}\right)$ and $\left(\dfrac{\pi}{4},\ \dfrac{\pi}{2}\right)$.

If $k<f\left(\dfrac{\pi}{4}\right)$, then $f(x) = k$ does not have any solution in $\left(0,\ \dfrac{\pi}{2}\right)$.

$\therefore$ $f(x) = k$ has 3 solutions only if

$k>f\left(\dfrac{\pi}{4}\right)\ \Rightarrow\ k>2\sqrt{2}$

(28 – 30) : 28. (d) 29. (c) 30. (c)

Let $\vec{r}=x\hat{i}+y\hat{j}$, $x^2 + y^2 + 8x - 10y + 40 = 0$,

centre $C(-4, 5)$, radius $= 1$, $p_1 = \max\{(x + 2)^2 + (y - 3)^2\}$

$p_2 = \min\{(x + 2)^2 + (y - 3)^2\}$

Let $P \equiv (-2, 3)$ and M be any point on circle.

$CP=2\sqrt{2},\ CM=1,\ p_1=(CP+CM)^2=(2\sqrt{2}+1)^2,$

$p_2=(CP-CM)^2=(2\sqrt{2}-1)^2, p_1 + p_2 = 18$

Now, slope of $AB=\left(\dfrac{dy}{dx}\right)_{(2,\ 2)}=-2$

Equation of AB is $2x + y = 6$

$\overrightarrow{OA}=2\hat{i}+2\hat{j},\ \overrightarrow{OB}=3\hat{i},\ \overrightarrow{AB}=\hat{i}-2\hat{j},$

$\overrightarrow{AB}\cdot\overrightarrow{OB}=(\hat{i}-2\hat{j})\,(3\hat{i})=3$

31. (P) → (2), (Q) → (1), (R) → (4), (S) → (3)

(P) $y=\tan^{-1}\dfrac{1}{2}+\tan^{-1}b,\quad (0<b<1)$

$0<\tan^{-1}\left(\dfrac{1+2b}{2-b}\right)\le\dfrac{\pi}{4}\ \Rightarrow\ 0<\left(\dfrac{1+2b}{2-b}\right)\le 1$

$\Rightarrow\ 0<(1+2b)\le(2-b)$

$\Rightarrow\ 3b\le 1\ \Rightarrow\ b\le\dfrac{1}{3}\ \Rightarrow\ b_{\max}=\dfrac{1}{3}$

(Q) $\sin^4 x + \cos^3 x \geq 1$ ...(i)

Since $\sin^2 x + \cos^2 x = 1$ and $-1 \leq \sin x, \cos x \leq 1$

(i) can not be > 1

$\therefore$ $\sin^4 x + \cos^3 x = \pm 1$ ...(ii)

(ii) is possible if either, $\sin x = \pm 1$ and $\cos x = 0$ or $\sin x = 0$ and $\cos x = 1$

$\Rightarrow\ x=(4n+1)\dfrac{\pi}{2},\ x=(2n+1)\dfrac{\pi}{2}$ or $x=n\pi,\ x=2n\pi$

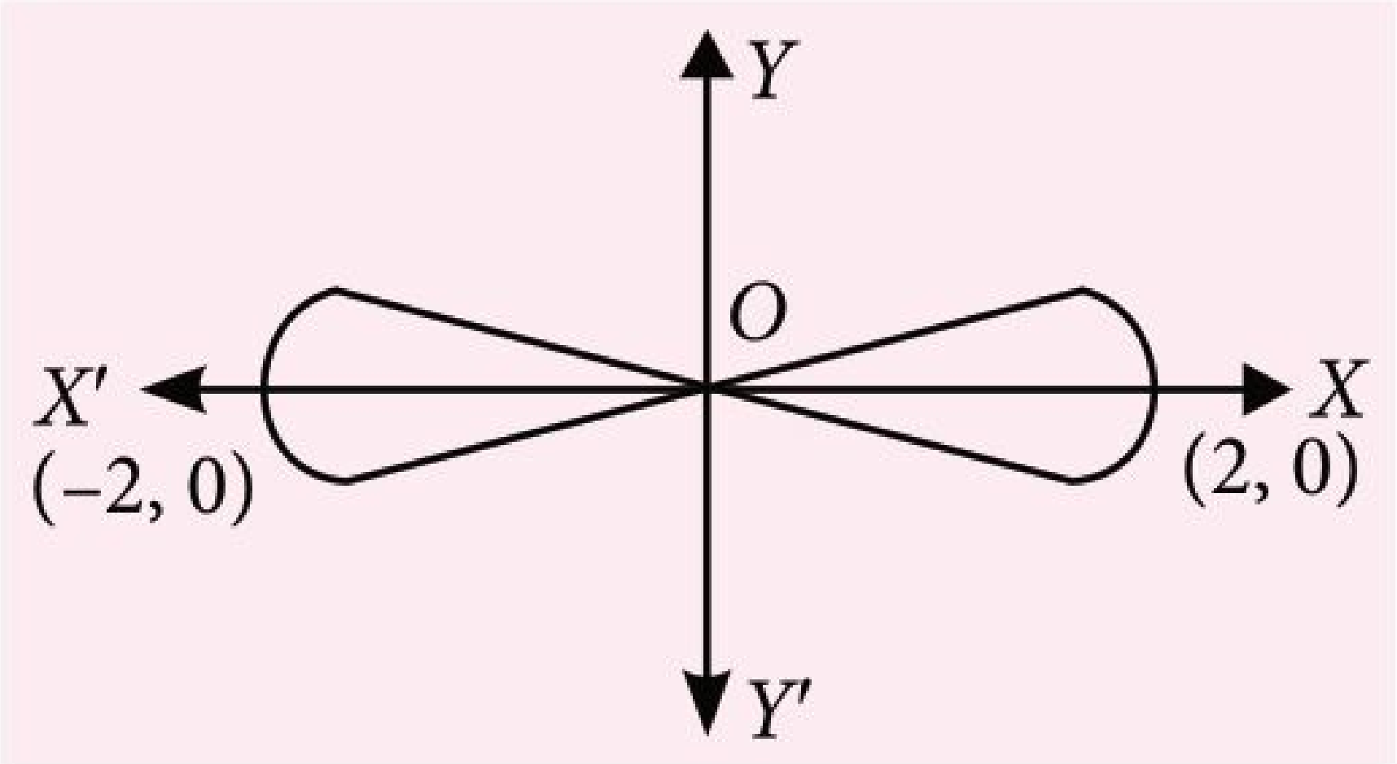
$\text{In}\,(0,\,2\pi), x=\dfrac{\pi}{2},\dfrac{3\pi}{2}$

$\therefore$ Number of solutions will be 2.

(R) $4y^2 = x^2(4 - x^2)$

$$\Rightarrow\ y=\frac{1}{2}\sqrt{x^2(4-x^2)}\ \Rightarrow y=\frac{x}{2}\sqrt{(4-x^2)}$$

$$\therefore\ \text{Area }(A)=4\times\int_0^2\frac{2x}{2\times 2}\sqrt{(4-x^2)}\,dx$$



Put $(4-x^2)=t\ \Rightarrow -2xdx=dt$

$$\therefore\ A=\frac{-4}{4}\int_4^0\sqrt{t}\,dt=\int_0^4\sqrt{t}\,dt=\left[\frac{t^{3/2}}{3/2}\right]_0^4=\frac{2}{3}\times[\sqrt{64}-0]$$

$$\Rightarrow\ A=\frac{16}{3}\text{ sq. units.}$$

(S) We have, $(1+x^2)\frac{dy}{dx}+2xy=1 \Rightarrow \frac{d}{dx}[(1+x^2)y]=1$

$\Rightarrow$ $(1+x^2)y=x+c$, Since $y(0)=0$

$\Rightarrow$ $(1+0)0=0+c\ \Rightarrow\ c=0$

$$\therefore\ (1+x^2)y=x \Rightarrow y=\frac{x}{(1+x^2)} \Rightarrow y(1)=\frac{1}{1+1}=\frac{1}{2}$$

32. (P) → (2, 4), (Q) → (3), (R) → (2), (S) → (5)

(P) The equation is $y=px+\frac{1}{p}$, where $p=\frac{dy}{dx}\neq 0$,

Differentiating w.r.t. x, we get

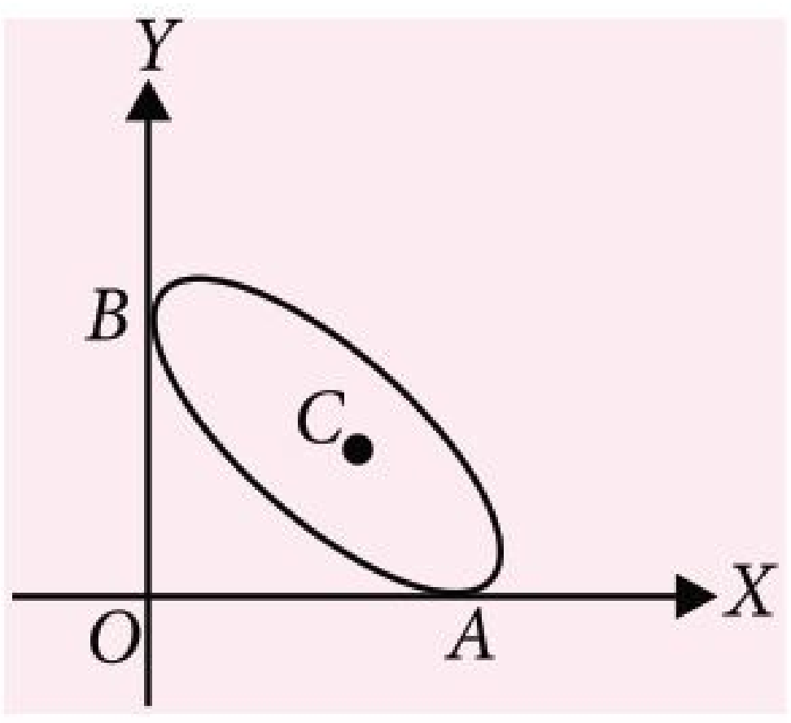
$$p=p+x\frac{dp}{dx}-\frac{1}{p^2}\frac{dp}{dx}\ \Rightarrow\ p^2=\frac{1}{x}\ \text{ or }\ \frac{dp}{dx}=0$$

Now, $\frac{dp}{dx}=0\ \Rightarrow\ p=\text{constant}=c$

$\therefore$ Equation is $y=cx+\frac{1}{c}$, which is a straight line.

If $p^2=\frac{1}{x}$, then $y^2=\frac{1}{x}\cdot x^2+x+2x=4x$, which is a parabola.

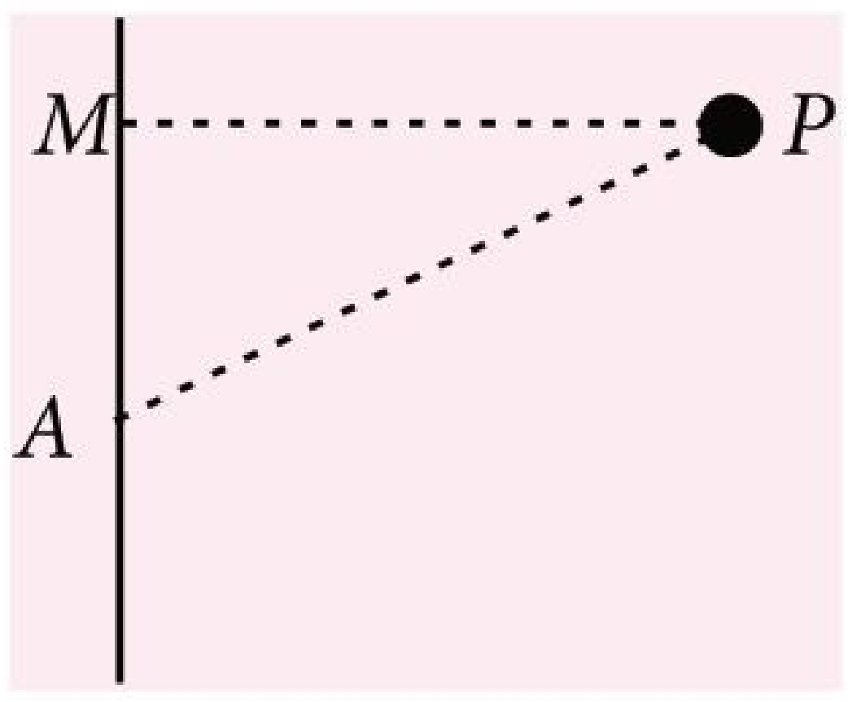
(Q) OA and OB are perpendicular tangents so O lies on the director circle and thus $OC^2=a^2+b^2$, where $2a$ is major axis and $2b$ the minor axis. Clearly, C lies on a circle.



(R) Given, $|ay-bx|=c\sqrt{(x-a)^2+(y-b)^2}$

$$\Rightarrow\ \frac{|ay-bx|}{\sqrt{a^2+b^2}}=\frac{c}{\sqrt{a^2+b^2}}\sqrt{(x-a)^2+(y-b)^2}$$

or $PM=kPA$, where PM is the length of perpendicular from $P(x,y)$ on the line $ay-bx=0$ and PA is the length of line segment joining P to the point $A(a,b)$ and A lies on the line, so the locus of P is a straight line through A.



(S) $hxy+ax+by+c=0 \Rightarrow\ hx\left(y+\frac{a}{h}\right)+b\left(y+\frac{c}{b}\right)=0$

Above can be converted into two factors. If $\frac{a}{h}=\frac{c}{b}$, then it will represent a pair of straight lines otherwise a rectangular hyperbola.

33. (P) → (3), (Q) → (2), (R) → (1), (S) → (1)

(P) $\vec{a},\vec{b}$ are unit vectors.

$$(\vec{a}+2\vec{b})\cdot(5\vec{a}-4\vec{b})=0\ \Rightarrow 5|\vec{a}|^2+6\cdot\vec{a}\cdot\vec{b}-8|\vec{b}|^2=0$$

$$\Rightarrow\ 6\vec{a}\cdot\vec{b}=3\ \Rightarrow\ \vec{a}\cdot\vec{b}=\frac{1}{2}$$

$$\Rightarrow\ 2(\vec{a}\cdot\vec{b})=1$$

(Q) Let $\vec{a}=(1,0,3),\ \vec{b}=(-1,3,4),\ \vec{c}=(1,2,1)$

and $\vec{d}=(k,2,5)$

a,b,c,d are coplanar if $[\vec{d}\,\vec{b}\,\vec{c}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{a}\,\vec{b}]=[\vec{a}\,\vec{b}\,\vec{c}]$

$$\Rightarrow \begin{vmatrix}k&2&5\\-1&3&4\\1&2&1\end{vmatrix}+\begin{vmatrix}k&2&5\\1&2&1\\1&0&3\end{vmatrix}+\begin{vmatrix}k&2&5\\1&0&3\\-1&3&4\end{vmatrix}=\begin{vmatrix}1&0&3\\-1&3&4\\1&2&1\end{vmatrix}$$

$\Rightarrow$ $k=-1$

(R) $(1,1,m),\ (1,1,m+1),\ (1,-1,m)$ are coplanar if

$$\begin{vmatrix}1&1&m\\1&1&m+1\\1&-1&m\end{vmatrix}=0\ \Rightarrow \begin{vmatrix}1&0&m\\1&0&m+1\\1&-2&m\end{vmatrix}=0$$

$\Rightarrow$ $2=0$ which is absurd.

$\therefore$ There is no value of m for which the vectors are coplanar.

(S) $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$,

$\vec{b}\times(\vec{c}\times\vec{a})=(\vec{b}\cdot\vec{a})\vec{c}-(\vec{c}\cdot\vec{b})\vec{a}$

and $\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}\cdot\vec{b})\vec{a}-(\vec{c}\cdot\vec{a})\vec{b}$

Since dot product is commutative, so we have, the sum = 0

34. (P) → (2), (Q) → (3), (R) → (1), (S) → (4)

(P) $3\cdot 1-2(-2)+5(\lambda)=0 \Rightarrow \lambda=-\frac{7}{5}$

(Q) Point $(3, \lambda, \mu)$ lies on $2x+y+z-3=x-2y+z-1$

$\Rightarrow \quad 3\cdot 2+\lambda+\mu-3=0$ and $3-2\lambda+\mu-1=0$

$\Rightarrow \quad \lambda+\mu+3=0$ and $2\lambda-\mu-2=0$ so $\lambda+\mu=-3$

(R) $\sin\theta=\dfrac{1\cdot 4+1(-3)+1\cdot 5}{\sqrt{1^2+1^2+1^2}\sqrt{16+9+25}}=\dfrac{6}{\sqrt{3}\sqrt{50}}$

$$\Rightarrow \theta=\sin^{-1}\sqrt{\frac{6}{25}}$$

(S) $\cos\theta=\dfrac{1\cdot 3+1(-4)+1\cdot 5}{\sqrt{3}\sqrt{16+9+25}}=\dfrac{4}{\sqrt{3}\sqrt{50}} \Rightarrow \theta=\cos^{-1}\sqrt{\dfrac{8}{75}}$

35. (P) → (5), (Q) → (2), (R) → (4), (S) → (1)

(P) Required number of arrangements $=\dfrac{6!}{2!\,3!}-20$
$= 60 - 20 = 40$

(Q) Coeff. of $\dfrac{T_{r+1}}{T_r}=\dfrac{{}^nC_r}{{}^nC_{r-1}}=\dfrac{n-r+1}{r}$ put $r=1, 2, ...$

$$\frac{C_1}{C_0}=\frac{n}{1}, \frac{C_2}{C_1}=\frac{n-1}{2}...,$$

Now, $\left(1+\dfrac{C_1}{C_0}\right).....\left(1+\dfrac{C_n}{C_{n-1}}\right)=\left(1+\dfrac{n}{1}\right)\left(1+\dfrac{n-1}{2}\right)...n$ factors

$=\dfrac{n+1}{1}\,\dfrac{n+1}{2}...n$ factors $=\dfrac{(n+1)^n}{n!}$

(R) Applying $R_1 \rightarrow R_1 + R_3 - 2R_2$, we get

$$\Delta=\begin{vmatrix} 0 & 0 & c+e-2d \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}$$

Given c, d, e are in A.P. $\therefore$ $2d = c + e$

$$\therefore \quad \begin{vmatrix} 0 & 0 & 0 \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}=0$$

(S) $c^2+d^2 \ge 2cd,\ d^2+e^2 \ge 2de,\ c^2+e^2 \ge 2ce$

[$\because$ A.M. ≥ G.M.]

$$\frac{c^2+d^2+e^2}{cd+de+ce} \ge 1$$

Also in ΔCDE, $d^2+e^2-c^2=2de\cos C<2de$,

$c^2+d^2-e^2=2cd\cos E<2cd$,

$c^2+e^2-d^2=2ce\cos D<2ce$

Adding, we get

$$c^2+d^2+e^2<2(ce+cd+de) \Rightarrow \frac{c^2+d^2+e^2}{ce+de+cd}<2$$

36. (4) : $f(3)=3f(1)=3, f(4)=f(2+1+1)$

$=2+1+1=4$ and so on.

In general, we get $f(r) = r$ for $r \in N$.

$$\Rightarrow \lim_{n\to\infty}\frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3}=\lim_{n\to\infty}\frac{12n(n+1)(2n+1)}{6n^3}=4$$

37. (2) : Put $x=r\cos\theta,\ y=r\sin\theta$.

$\therefore \quad r^2\sin\theta\cos\theta = 1 - r^2 \Rightarrow r^2=\dfrac{2}{2+\sin 2\theta}, \ \dfrac{2}{3}\le r^2 \le 2$

Let $k=x^3y+xy^3=r^2(1-r^2)$ and $r^2=t, \dfrac{2}{3}\le t\le 2$

So, $k = (1 - t)\,t$

$\Rightarrow \quad \dfrac{dk}{dt}=1-2t<0$ for $\dfrac{2}{3}\le t\le 2$

So, k is min when t is max *i.e.* for $t = 2$, k is min.

When $t = 2$, min k is -2.

$\therefore \quad x^3y + xy^3 + 4 \ge 2$

38. (3) : $\dfrac{(x-4)^2}{16}+\dfrac{(y-3)^2}{9}=1$ and point P on the curve $4(1 + \cos\theta), 3(1 + \sin\theta)$.

The reflection of the point P about the line $x - y - 2 = 0$ can be found using

$$\frac{h-4(1+\cos\theta)}{1}=\frac{k-3(1+\sin\theta)}{-1}$$

$$=\frac{-2(4(1+\cos\theta)-3(1+\sin\theta)-2)}{2}=-(4\cos\theta-3\sin\theta)+1$$

$h = 4(1 + \cos\theta) - 4\cos\theta + 3\sin\theta + 1 = 4 + 3\sin\theta + 1$

and $k = 3(1 + \sin\theta) + 4\cos\theta - 3\sin\theta - 1 = 3 + 4\cos\theta - 1$

$$\left(\frac{h-5}{3}\right)^2+\left(\frac{k-2}{4}\right)^2=1$$

$\therefore$ Equation of image is $16(x - 5)^2 + 9(y - 2)^2 = 144$

$\Rightarrow 16x^2 + 9y^2 - 160x - 36y + 292 = 0$

$\therefore \quad k_1 = -160, k_2 = 292 \quad \Rightarrow \dfrac{k_1+k_2}{44}=\dfrac{132}{44}=3$

39. (2) **40. (1)**

◆◆

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If a, b, c are in G.P. and the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(Priyansh Sharma, Delhi)

Ans. We have, $b^2 = ac$ ...(i)

The equation $ax^2 + 2bx + c = 0$ gives

$$x = \frac{-2b \pm \sqrt{4b^2 - 4ac}}{2a} = \frac{-b}{a} \qquad \text{[Using (i)]}$$

and $dx^2 + 2ex + f = 0 \Rightarrow x = \dfrac{-e \pm \sqrt{e^2 - df}}{d}$

According to the given condition, we have

$$\frac{-b}{a} = \frac{-e \pm \sqrt{e^2 - df}}{d} \quad \text{i.e.,} \quad \left(\frac{e}{d} - \frac{b}{a}\right)^2 = \frac{e^2 - df}{d^2}$$

$$\text{i.e.,} \quad \frac{2be}{ad} = \frac{b^2}{a^2} + \frac{f}{d} = \frac{c}{a} + \frac{f}{d} \qquad \text{[Using (i)]}$$

$$\text{i.e.,} \quad \frac{2be}{ac} = \frac{d}{a} + \frac{f}{c} \qquad \left[\text{Multiplying both sides by } \frac{d}{c}\right]$$

$$\text{i.e.,} \quad \frac{2e}{b} = \frac{d}{a} + \frac{f}{c} \qquad \text{[Using (i)]}$$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

2. Find the values of a, b, c if the following limit exists :

$$\lim_{x \to 0} \frac{ae^x - b\ln(1+x) + cxe^{-x}}{x^2 \sin x} = 2$$

(Rajnish, Jharkhand)

Ans. Let $L = \displaystyle\lim_{x \to 0} \frac{ae^x - b\ln(1+x) + cxe^{-x}}{x^2 \sin x}$

$$= \lim_{x \to 0} \frac{ae^x + cxe^{-x} - b\ln(1+x)}{x^3} \cdot \frac{x}{\sin x}$$

$$= \lim_{x \to 0} \frac{ae^x + cxe^{-x} - b\ln(1+x)}{x^3} \qquad \left[\because \lim_{x \to 0} \frac{x}{\sin x} = 1\right]$$

$$= \lim_{x \to 0} \frac{a\left(1 + x + \frac{x^2}{2!} + \ldots\right) + cx\left(1 - x + \frac{x^2}{2!} - \ldots\right) - b\left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{a + (a+c-b)x + (a-2c+b)\frac{x^2}{2} + \left(\frac{a}{3!} + \frac{c}{2!} - \frac{b}{3}\right)x^3 + \ldots}{x^3}$$

For a finite non-zero limit to exist, we have

$a = 0$ ...(i)

$a + c - b = 0$ ...(ii)

$a - 2c + b = 0$ ...(iii)

and $\frac{a}{6} + \frac{c}{2} - \frac{b}{3} = 2$ [according to the given condition]

i.e., $a + 3c - 2b = 12$ ...(iv)

Solving (i), (ii), (iii) and (iv), we can see that there is no value of a, b, c which satisfies all the equations.

3. Consider the function

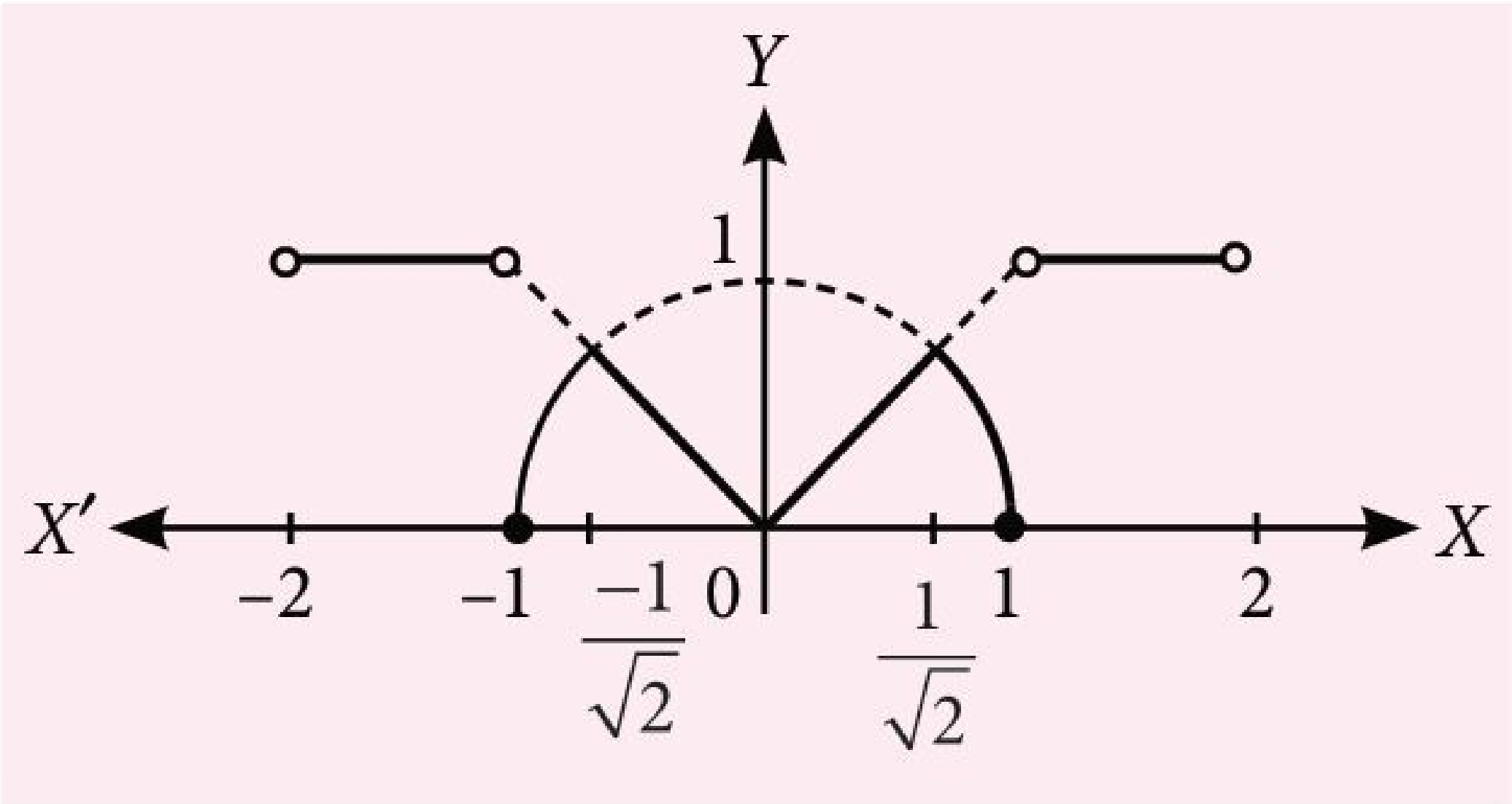
$$f(x) = \min\left\{|x|, \sqrt{1-x^2}\right\}, -1 \le x \le 1$$
$$= [|x|], 1 \le |x| \le 2$$

Plot the curve and discuss its continuity and differentiability from the graph.

(Ritu Jain, Gujarat)

Ans. The given figure shows the curves

$y_1 = |x|, \quad -1 \le x \le 1$



$y_2 = \sqrt{1-x^2}, -1 \le x \le 1$

Which is the upper half of the circle $x^2 + y^2 = 1$

and $y_3 = [|x|], x \in (-2, -1) \cup (1, 2)$

The portion shown in thicker strokes, represents the curve

$$y = \min\left\{|x|, \sqrt{1-x^2}\right\}, -1 \le x \le 1$$
$$= [|x|], 1 < |x| < 2$$

From the graph, we can see that $f(x)$ is continuous everywhere on $x \in (-2, 2) - \{-1, 1\}$

and differentiable everywhere on

$x \in (-2, 2) - \left\{\pm 1, \frac{\pm 1}{\sqrt{2}}, 0\right\}$.

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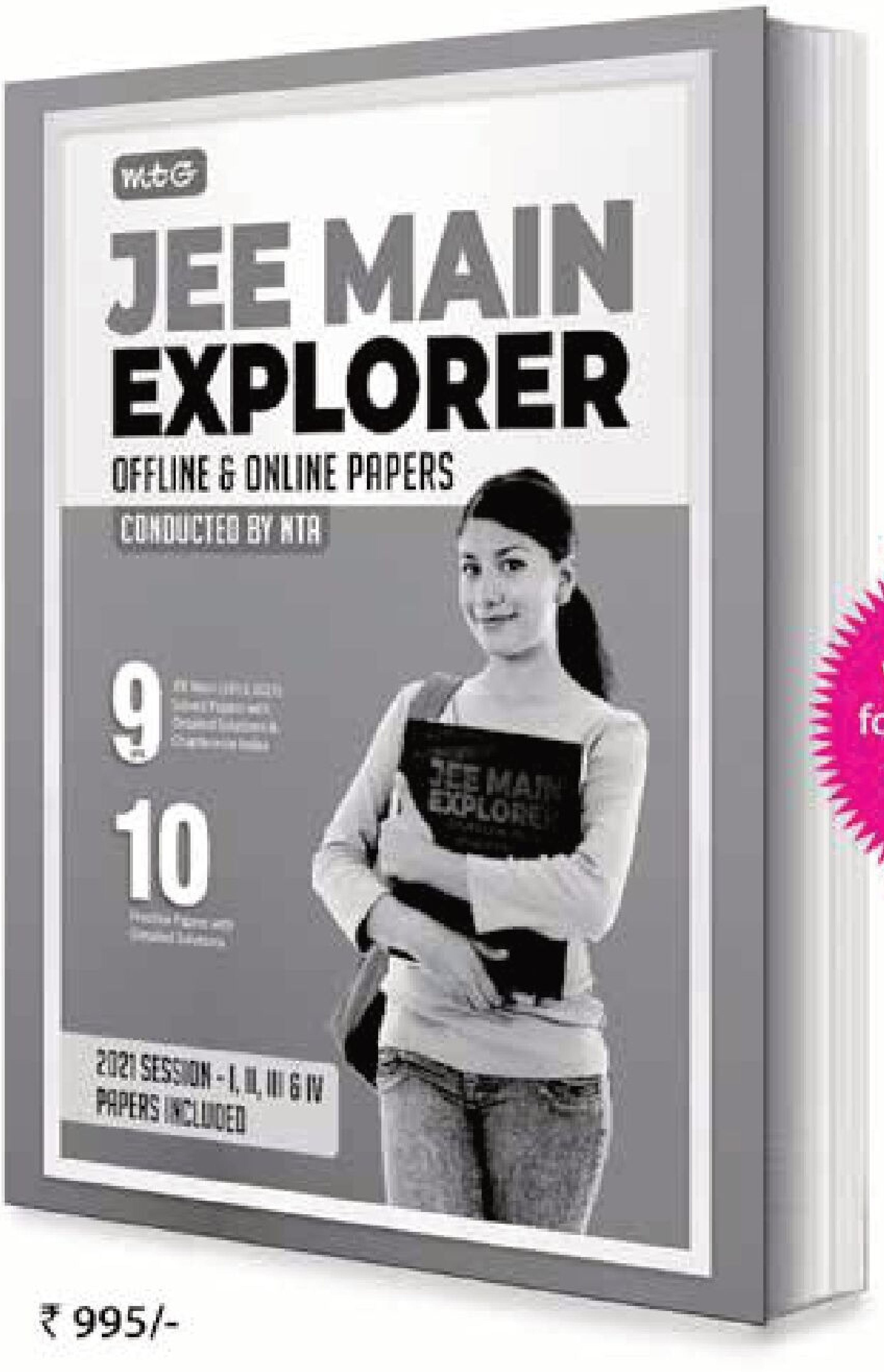
What is JEE (MAIN) like?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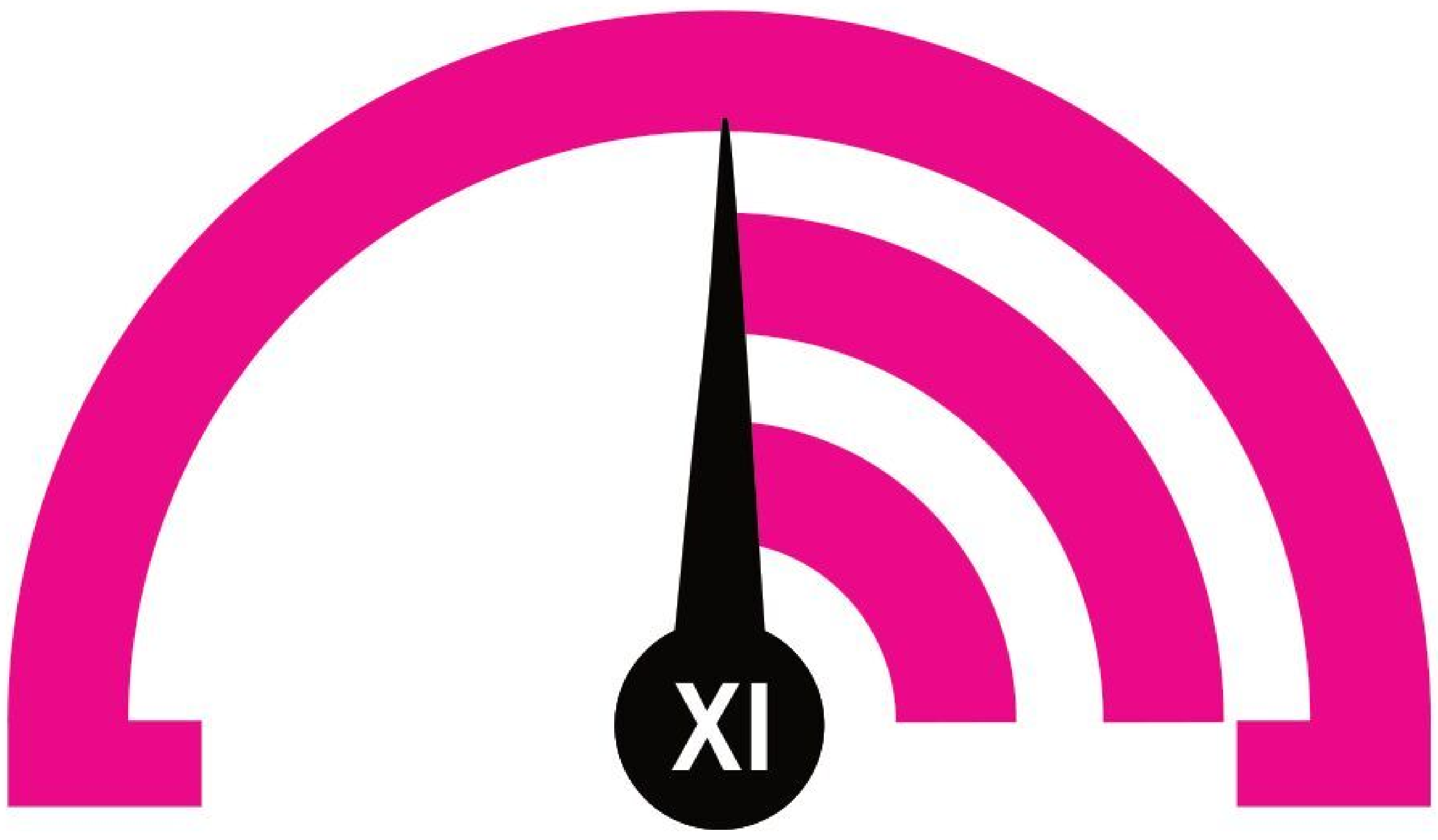
MONTHLY TEST DRIVE

Practice Paper



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The solution of the equation $(3|x| - 3)^2 = |x| + 7$ which belongs to the domain of $\sqrt{x(x-3)}$ are given by

(a) $\pm\frac{1}{9}, \pm 2$ (b) $\frac{1}{9}, -3$

(c) $-\frac{1}{6}, -2$ (d) $-\frac{1}{9}, 0$

2. In a G.P. if the sum of infinite terms is 20 and the sum of their squares is 100, then the common ratio is

(a) 5 (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

3. Let S be a non-empty subset of R. Consider the following statement:

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(a) There is a rational number $x \in S$ such that $x \leq 0$.

(b) There is no rational number $x \in S$ such that $x \leq 0$.

(c) Every rational number $x \in S$ satisfies $x \leq 0$.

(d) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational.

4. Six boys and six girls sit in a row randomly. The probability that the six girls sit together or the boys and girls sit alternately is

(a) $\frac{3}{308}$ (b) $\frac{1}{100}$ (c) $\frac{2}{205}$ (d) $\frac{4}{407}$

5. The general solution of the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is

(a) $n\pi + \frac{\pi}{4}$ (b) $\frac{n\pi}{2} + \frac{\pi}{4}$

(c) $n\pi + \frac{\pi}{8}$ (d) $\frac{n\pi}{2} + \frac{\pi}{8}$

6. Let $P(n) = 3^{2n}\ \forall\ n \in N$, when divided by 8, leaves the remainder

(a) 2 (b) 1 (c) 4 (d) 7

One or More Than One Option(s) Correct Type

7. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ (where $i = \sqrt{-1}$) is a real number is

(a) $2n\pi + \frac{\pi}{2}, n \in I$ (b) $2n\pi - \frac{\pi}{2}, n \in I$

(c) $2n\pi \pm \frac{\pi}{2}, n \in I$ (d) $2n\pi \pm \frac{\pi}{4}, n \in I$

8. The number of ways of arranging seven persons (having A, B, C and D among them) in a row so that A, B, C and D are always in order A-B-C-D (not necessarily together) is

(a) 210 (b) 5040 (c) $6 \times {}^7C_4$ (d) ${}^7P_3$

9. If n is a positive integer and $(3\sqrt{3}+5)^{2n+1} = \alpha + \beta$, where α is an integer and $0 < \beta < 1$, then

(a) α is an even integer

(b) α is divisible by 10

(c) the integer just below $(3\sqrt{3}+5)^{2n+1}$ divisible by 3

(d) None of these

10. If the lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$ and $\lambda x + 4y + \lambda^2 = 0$ are concurrent, then

(a) $\lambda = 2$ (b) $\lambda = -3$

(c) $\lambda = 4$ (d) $\lambda = -4$

11. The equation $\sin x = [1 + \sin x] + [1 - \cos x]$ has (where $[x]$ is the greatest integer less than or equal to x)

(a) no solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) no solution in $\left[\frac{\pi}{2}, \pi\right]$

(c) no solution in $\left[\pi, \frac{3\pi}{2}\right]$

(d) None of these

12. In a ΔABC, $\tan A$ and $\tan B$ satisfy the inequation $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then

(a) $a^2 + b^2 + ab > c^2$ (b) $a^2 + b^2 - ab < c^2$

(c) $a^2 + b^2 > c^2$ (d) none of these

13. Let $f(x) = \dfrac{5\sqrt{\sin x}}{1 + \sqrt[3]{\sin x}}$. If D is the domain of f, then D contains

(a) $(0, \pi)$ (b) $(-2\pi, -\pi)$

(c) $(2\pi, 3\pi)$ (d) $(4\pi, 6\pi)$

Comprehension Type

Paragraph for Q. No. 14 to 15

ABC is a triangle right angled at A, $AB = 2AC$. $A = (1, 2)$, $B = (-3, 1)$. ACD is an equilateral triangle. The vertices of the two triangles are in anti-clockwise sense. $BCEF$ is a square with vertices in clockwise sense.

14. $\Delta ACF =$

(a) $\frac{51}{8}$ (b) $\frac{51}{4}$

(c) $\frac{31}{5}$ (d) $\frac{21}{4}$

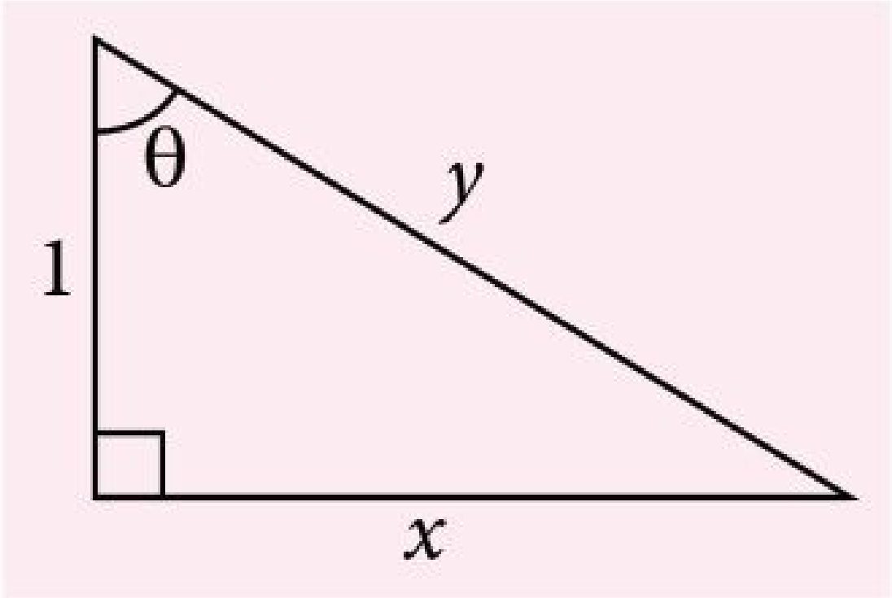
15. $DE =$

(a) $\sqrt{17}\sqrt{4-\sqrt{3}}$ (b) $\frac{\sqrt{17}}{2}\sqrt{8+\sqrt{3}}$

(c) $\frac{\sqrt{17}}{2}\sqrt{4+\sqrt{3}}$ (d) $\sqrt{15}\sqrt{4+\sqrt{3}}$

Matrix Match Type

16. A right angled triangle has sides 1 and x. The hypotenuse is y and the angle opposite to the side x is θ.



	Column-I		Column-II
P.	$\lim\limits_{\theta \to \frac{\pi}{2}} \sqrt{y} - \sqrt{x}$	(1)	0
Q.	$\lim\limits_{\theta \to \frac{\pi}{2}} y - x$	(2)	$\frac{1}{2}$
R.	$\lim\limits_{\theta \to \frac{\pi}{2}} y^2 - x^2$	(3)	1
S.	$\lim\limits_{\theta \to \frac{\pi}{2}} y^3 - x^3$	(4)	∞

	P	Q	R	S
(a)	4	3	2	1
(b)	1	1	3	4
(c)	1	2	3	4
(d)	2	2	4	3

Numerical Answer Type

17. $\sin^2 12° + \sin^2 21° + \sin^2 39° + \sin^2 48° - \sin^2 9° - \sin^2 18°$ is

18. If α is a complex seventh root of unity, then the equation whose roots are $\alpha + \alpha^2 + \alpha^4$ and $\alpha^3 + \alpha^5 + \alpha^6$ is $x^2 + x + c = 0$, where c is

19. If the integer k is added to each of the numbers, 36, 300, 596, one obtains the squares of three consecutive terms of an A.P., then the last digit of k is

20. If $\log_{245} 175 = a$, $\log_{1715} 875 = b$, then the value of $\dfrac{1-ab}{a-b}$ is

⁂

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CBSE TERM-II

Held on 7th June

SOLVED PAPER 2022

CLASS 12

General Instructions :

Time allowed : 2 hours

Maximum Marks : 40

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains three sections -Section A, B and C.

(ii) Each section is compulsory.

(iii) Section A has 6 short answer type I questions of 2 marks each.

(iv) Section B has 4 short answer type II questions of 3 marks each.

(v) Section C has 4 long answer type questions of 4 marks each.

(vi) There is an internal choice in some questions.

(vii) Question no. 14 is a case-study based question with 2 sub-parts of 2 marks each.

SECTION A

Questions number 1 to 6 carry 2 marks each.

1. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

2. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
$P(X)$	K	$6K$	$9K$

(a) Find the value of K.

(b) Is the coin tossed biased or unbiased? Justify your answer.

3. **(a)** If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=400$ and $|\vec{b}|=5$, then find the value of $|\vec{a}|$.

OR

(b) Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes.

4. Find the general solution of the differential equation

$\sec^2 x\cdot\tan y\,dx+\sec^2 y\cdot\tan x\,dy=0$.

5. Evaluate : $\int_0^1 x^2e^x\,dx$

6. There are two bags. Bag I contains 1 red and 3 white balls, and Bag II contains 3 red and 5 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball so drawn is red in colour.

SECTION B

Questions number 7 to 10 carry 3 marks each.

7. Using integration, find the area of the region $\{(x, y) : y^2 \le x \le y\}$.

8. **(a)** If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

OR

(b) Check whether the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are skew or not.

9. **(a)** Find : $\int\frac{1}{e^x+1}dx$

OR

(b) Evaluate : $\int_1^4\{|x|+|3-x|\}dx$

10. If $\vec{a}$ and $\vec{b}$ are two vectors of equal magnitude and α is the angle between them, then prove that $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|}=\cot\left(\frac{\alpha}{2}\right)$.

SECTION C

Questions number 11 to 14 carry 4 marks each.

11. **(a)** Find the particular solution of the differential equation $x\frac{dy}{dx}+y+\frac{1}{1+x^2}=0,$ given that $y(1) = 0$.

OR

(b) Find the general solution of the differential equation $x(y^3 + x^3)\, dy = (2y^4 + 5x^3y)dx$.

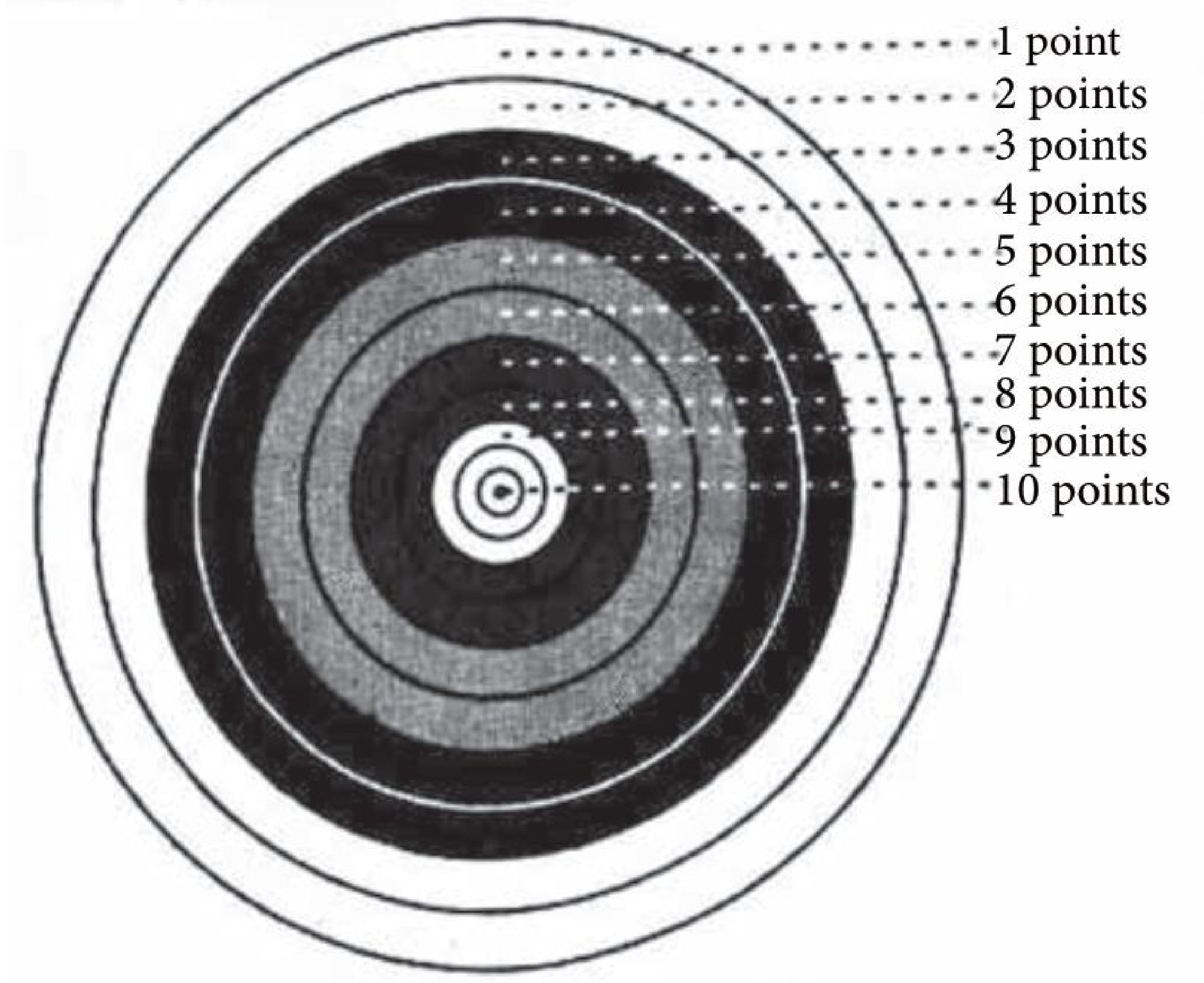
12. Evaluate : $\int_0^{\pi}\frac{x}{9\sin^2 x+16\cos^2 x}dx$

13. Find the equations of the planes passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j})=6$ and $\vec{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})=0,$ which are at a distance of 1 unit from the origin.

Case-Study Based Question

14. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely to earn 10 points with a probability of 0.9.



Based on the above information, answer the following questions :

If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points.

(b) both of them earn 10 points.

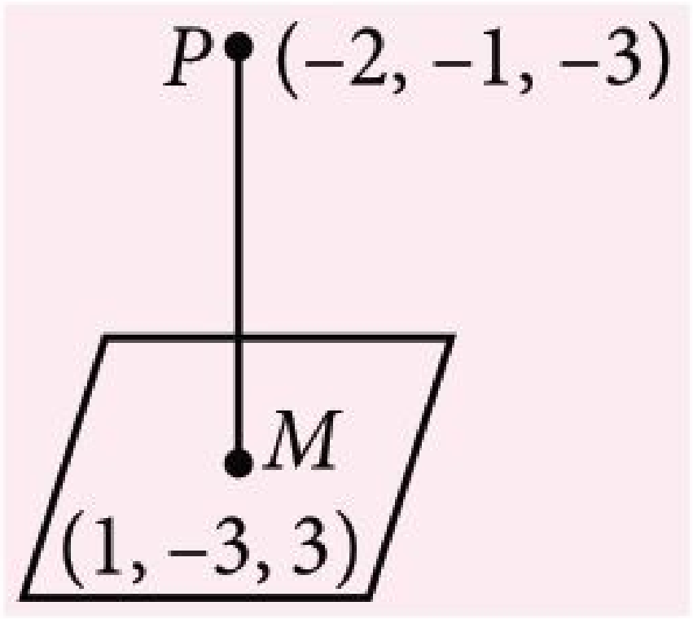
SOLUTIONS

1. Let point P be $(-2, -1, -3)$ and M be $(1, -3, 3)$.

Clearly, PM is perpendicular to the plane.

D.r's of PM are $<1 - (-2), -3 - (-1), 3 - (-3)>$ *i.e.*, $<3, -2, 6>$

$\therefore$ Equation of plane passing through point $M(1, -3, 3)$ and d.r's of perpendicular is $<3, -2, 6>$ is given by



$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$\Rightarrow\ 3(x - 1) - 2(y + 3) + 6(z - 3) = 0$$

$$\Rightarrow\ 3x - 2y + 6z - 27 = 0$$

2. **(a)** We know $\Sigma p(x_i) = 1$

$$\Rightarrow\ K + 6K + 9K = 1$$

$$\Rightarrow\ 16K = 1 \Rightarrow K=\frac{1}{16}$$

(b) P(getting 2 heads) $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ (if the coin was unbiased)

But from given p.d. table, P(getting 2 heads) $=\frac{1}{16}\neq\frac{1}{4}$

$\therefore$ Coin tossed is biased.

3. **(a)** We have, $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=400$

$$\Rightarrow\ |\vec{a}|^2|\vec{b}|^2\sin^2\theta+|\vec{a}|^2|\vec{b}|^2\cos^2\theta=400$$

$$\Rightarrow\ |\vec{a}|^2|\vec{b}|^2=400 \qquad (\because \sin^2\theta+\cos^2\theta=1)$$

$$\Rightarrow\ |\vec{a}|^2\times 25=400 \Rightarrow |\vec{a}|^2=\frac{400}{25}=16 \Rightarrow |\vec{a}|=4$$

OR

(b) $\pm 5\sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)$ are two possible vector of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes.

4. We have, $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$$\Rightarrow\ \frac{\sec^2 x}{\tan x}dx=-\frac{\sec^2 y}{\tan y}dy$$

$$\Rightarrow\ \frac{d(\tan x)}{\tan x}=-\frac{d(\tan y)}{\tan y}$$

$\Rightarrow$ $\log(\tan x) = -\log(\tan y) + \log c$ (integrating on both sides)

$\Rightarrow$ $\tan x \tan y = c$

5. $\int_0^1 x^2e^x dx = [x^2e^x]_0^1 - 2\int_0^1 xe^x dx$

$= e - 2[xe^x - e^x]_0^1 = e - 2(0-(-1)) = e - 2$

6. Let E_1 be the event that bag I is chosen,

E_2 be the event that bag II is chosen and A be the event that red ball is drawn.

Clearly, E_1 and E_2 are mutually exclusive and exhaustive events.

Since, one of the bag is chosen at random

$\therefore \quad P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$

$P(A \mid E_1) = \frac{1}{4}$ and $P(A \mid E_2) = \frac{3}{8}$

By using law of total probability, we get

$P(A) = P(E_1)\, P(A \mid E_1) + P(E_2)\, P(A \mid E_2)$

$= \frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{3}{8} = \frac{1}{8}+\frac{3}{16} = \frac{5}{16}$

7. On solving $x = y$ and $x = y^2$, we get

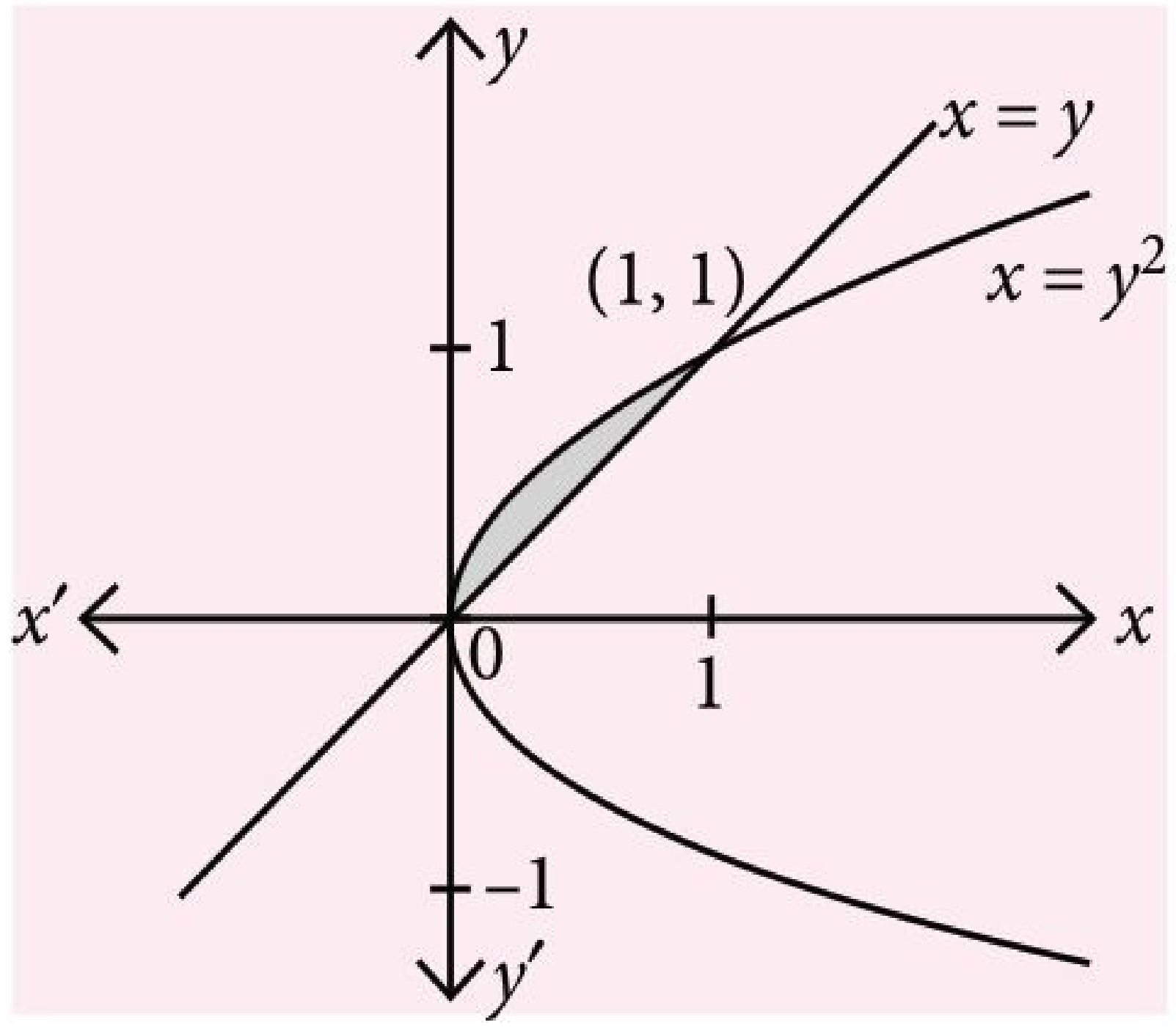
$y^2 = y \Rightarrow y(y-1) = 0 \Rightarrow y = 0$ and $y = 1$.

$\therefore$ Required area = area of shaded region

$= \int_0^1 (y - y^2)dy$

$= \left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1$

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ sq. unit



8. **(a)** Let θ be the angle made by the line with y-axis.

Then, $\cos^2 60° + \cos^2 45° + \cos^2\theta = 1$

$\Rightarrow \quad \frac{1}{4}+\frac{1}{2}+\cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \theta = 60°$

$\therefore$ Direction cosines of the line are

<cos60°, cos60°, cos45°> *i.e.*, $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$.

OR

(b) We know, skew lines are those lines which do not lie in the same plane.

We first compute $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ ...(i)

If (i) is equal to 0, then the lines are not skew line.

Here, $x_1 = 1, y_1 = 2, z_1 = 3, x_2 = 4, y_2 = 1, z_2 = 0$

$a_1 = 2, b_1 = 3, c_1 = 4, a_2 = 5, b_2 = 2, c_2 = 0$

$\therefore \quad \begin{vmatrix} 3 & -1 & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 0 \end{vmatrix} = -11 \neq 0$

$\therefore$ Given lines are skew lines.

9. **(a)** Let $I = \int \frac{1}{e^x + 1} dx$

Put $e^x + 1 = t \Rightarrow e^x\, dx = dt \Rightarrow dx = \frac{dt}{e^x} = \frac{dt}{t-1}$

$\therefore \quad I = \int \frac{dt}{t(t-1)} = \int \frac{t-(t-1)}{t(t-1)} dt$

$= \int \frac{1}{t-1} dt - \int \frac{1}{t} dt = \log(t-1) - \log t + C$

$= \log e^x - \log(e^x + 1) + c = \log \frac{e^x}{e^x + 1} + c$

OR

(b) Let $I = \int_1^4 \{|x| + |3 - x|\} dx$

$$|x| + |3-x| = \begin{cases} x + 3 - x, & 1 \le x < 2 \\ x + 3 - x, & 2 \le x < 3 \\ x + x - 3, & 3 \le x < 4 \end{cases} = \begin{cases} 3, & 1 \le x < 3 \\ 2x - 3, & 3 \le x < 4 \end{cases}$$

$\therefore \quad I = \int_1^3 3dx + \int_3^4 (2x-3)dx = [3x]_1^3 + \left[\frac{2x^2}{2} - 3x\right]_3^4$

= (9 –3) + (16 – 12 – 9 + 9)

= 6 + 4 = 10

10. We have, $|\vec{a}| = |\vec{b}|$

We have to prove, $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = \cot\frac{\alpha}{2}$

i.e., $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| \cot\frac{\alpha}{2}$

i.e, $|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2 \cot^2\frac{\alpha}{2}$

L.H.S. $= |\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

$= 2|\vec{a}|^2 + 2|\vec{a}||\vec{b}|\cos\alpha \quad (\because |\vec{a}| = |\vec{b}|)$

$= 2|\vec{a}|^2 + 2|\vec{a}|^2 \cos\alpha$

$$= 2|\vec{a}|^2 (1+\cos\alpha) = 2|\vec{a}|^2\, 2\cos^2\frac{\alpha}{2}$$

$$= 4|\vec{a}|^2 \cos^2\frac{\alpha}{2} \qquad \text{... (i)}$$

R.H.S. $= |\vec{a}-\vec{b}|^2 \cot^2\frac{\alpha}{2} = \left(|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}\right)\cot^2\left(\frac{\alpha}{2}\right)$

$$= (2|\vec{a}|^2 - 2|\vec{a}||\vec{b}|\cos\alpha)\cdot\cot^2\left(\frac{\alpha}{2}\right)$$

$$= 2|\vec{a}|^2 (1-\cos\alpha)\cot^2\frac{\alpha}{2} = 2|\vec{a}|^2 \cdot 2\sin^2\frac{\alpha}{2}\cdot\frac{\cos^2\alpha/2}{\sin^2\alpha/2}$$

$$= 4|\vec{a}|^2 \cos^2\alpha/2 \qquad \text{...(ii)}$$

∴ From (i) and (ii),

L.H.S. = R.H.S.

11. (a) We have, $x\frac{dy}{dx}+y+\frac{1}{1+x^2}=0$ ...(i)

$$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=\frac{-1}{(1+x^2)x}$$

Clearly, it is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P=\frac{1}{x}$ and $Q=\frac{-1}{x(1+x^2)}$

$$\text{I.F.} = e^{\int\frac{1}{x}dx} = e^{\log x} = x.$$

∴ Solution of (i) is

$$y\cdot x = \int\frac{-1}{x(1+x^2)}\cdot x\,dx + C$$

$$\Rightarrow yx = \int\frac{-1}{1+x^2}dx + C \Rightarrow yx = -\tan^{-1}x + C$$

Also, given $y(1) = 0$

∴ $0\times 1 = -\tan^{-1}1 + C \Rightarrow C = \tan^{-1}1 = \frac{\pi}{4}$

∴ Particular solution of given differential equation is

$$yx = -\tan^{-1}x + \frac{\pi}{4}$$

OR

(b) We have, $x(y^3+x^3)\,dy = (2y^4+5x^3y)dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2y^4+5x^3y}{xy^3+x^4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(y/x)^4+5(y/x)}{(y/x)^3+1}$$

Put $v=\frac{y}{x}\Rightarrow y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore v+x\frac{dv}{dx}=\frac{2v^4+5v}{v^3+1}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{2v^4+5v}{v^3+1}-v=\frac{v^4+4v}{v^3+1}$$

$$\Rightarrow \frac{v^3+1}{v^4+4v}dv=\frac{dx}{x} \Rightarrow \int\frac{v^3+1}{v^4+4v}dv=\int\frac{dx}{x}$$

Putting $v^4+4v=t \Rightarrow (4v^3+4)\,dv = dt$

$$\therefore \frac{1}{4}\int\frac{dt}{t}=\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{4}\log(v^4+4v)=\log x+\log C$$

$$\Rightarrow \frac{1}{4}\log(v^4+4v)=\log x+\log C$$

$$\Rightarrow \frac{y^4+4yx^3}{x^8}=C$$

12. Let $I=\int_0^\pi\frac{x}{9\sin^2x+16\cos^2x}dx$ ...(i)

$$\Rightarrow I=\int_0^\pi\frac{(\pi-x)dx}{9\sin^2(\pi-x)+16\cos^2(\pi-x)}$$

$$\left(\text{Using } \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^\pi\frac{(\pi-x)dx}{9\sin^2x+16\cos^2x} \qquad \text{...(ii)}$$

EXAM ALERT 2022

Exam	Date
JEE Main	Session 1: 23rd to 29th June Session 2: 21st to 30th July
BITSAT	Session-I: 2nd to 9th July Session-II: 3rd to 7th August
KEAM	4th July
VITEEE	30th June to 6th July
JEE Advanced	28th August

Adding (i) and (ii), we get

$$2I=\int_0^{\pi}\frac{\pi dx}{9\sin^2 x+16\cos^2 x}$$

Consider $f(x)=\dfrac{1}{9\sin^2 x+16\cos^2 x}$

$$f(\pi-x)=\frac{1}{9\sin^2(\pi-x)+16\cos^2(\pi-x)}$$

$$=\frac{1}{9\sin^2 x+16\cos^2 x}=f(x)$$

$$\therefore\ I=\pi\int_0^{\pi/2}\frac{dx}{9\sin^2 x+16\cos^2 x}$$

$$\left(\text{Using } \int_0^{2a} f(x)dx=2\int_0^{a} f(x)dx, \text{ if } f(2a-x)=f(x)\right)$$

$$\Rightarrow\ I=\pi\int_0^{\pi/2}\frac{\sec^2 x\,dx}{9\tan^2 x+16}\ \text{(Dividing } N^r \text{ \& } D^r \text{ by } \cos^2 x)$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

When $x = 0, t = 0; x = \pi/2, t = \infty$

$$\Rightarrow\ I=\pi\int_0^{\infty}\frac{dt}{9t^2+16}=\frac{\pi}{9}\int_0^{\infty}\frac{dt}{t^2+\frac{16}{9}}$$

$$=\frac{\pi}{9}\cdot\frac{3}{4}\left[\tan^{-1}\frac{3t}{4}\right]_0^{\infty}=\frac{\pi}{12}\left(\tan^{-1}\infty-\tan^{-1}0\right)$$

$$=\frac{\pi^2}{24}$$

13. Here, $\vec{n}_1=\hat{i}+3\hat{j},\ \vec{n}_2=3\hat{i}-\hat{j}-4\hat{k}$

$d_1 = 6, d_2 = 0$

Using the relation $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$, we get

$\vec{r}\cdot(\hat{i}+3\hat{j}+\lambda(3\hat{i}-\hat{j}-4\hat{k}))=6+\lambda\cdot 0$, where λ is some real number

$$\Rightarrow\ (x\hat{i}+y\hat{j}+z\hat{k})\cdot((1+3\lambda)\hat{i}+(3-\lambda)\hat{j}-4\lambda\hat{k})=6$$

$\Rightarrow\ (1 + 3\lambda)x + (3 - \lambda)y - 4\lambda z = 6$ ...(i)

Given that the distance of plane (i) from (0, 0, 0) is 1 unit.

$$\therefore\ \left|\frac{(0\hat{i}+0\hat{j}+0\hat{k})\cdot\left[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}-4\lambda\hat{k}\right]-6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}\right|$$

$$\Rightarrow\ \sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}=6$$

$\Rightarrow\ (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2 = 36$

$\Rightarrow\ 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2 = 36$

$\Rightarrow\ 26\lambda^2 = 26 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

$\therefore$ Required equations of planes are

$(1 + 3)x + (3 - 1)y - 4z = 6$ and $(1 - 3)x + (3 + 1)y + 4z = 6$

i.e., $4x + 2y - 4z = 6$ and $-2x + 4y + 4z = 6$.

14. We have, $P(A) = 0.8$, $P(B) = 0.9$

(a) P(exactly one of them earn 10 points)

$= P(A \cup B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

$= P(A) + P(B) - 2P(A)\,P(B)$ ($\because$ A & B are independent)

$= 0.8 + 0.9 - 0.8 \times 0.9 \times 2$

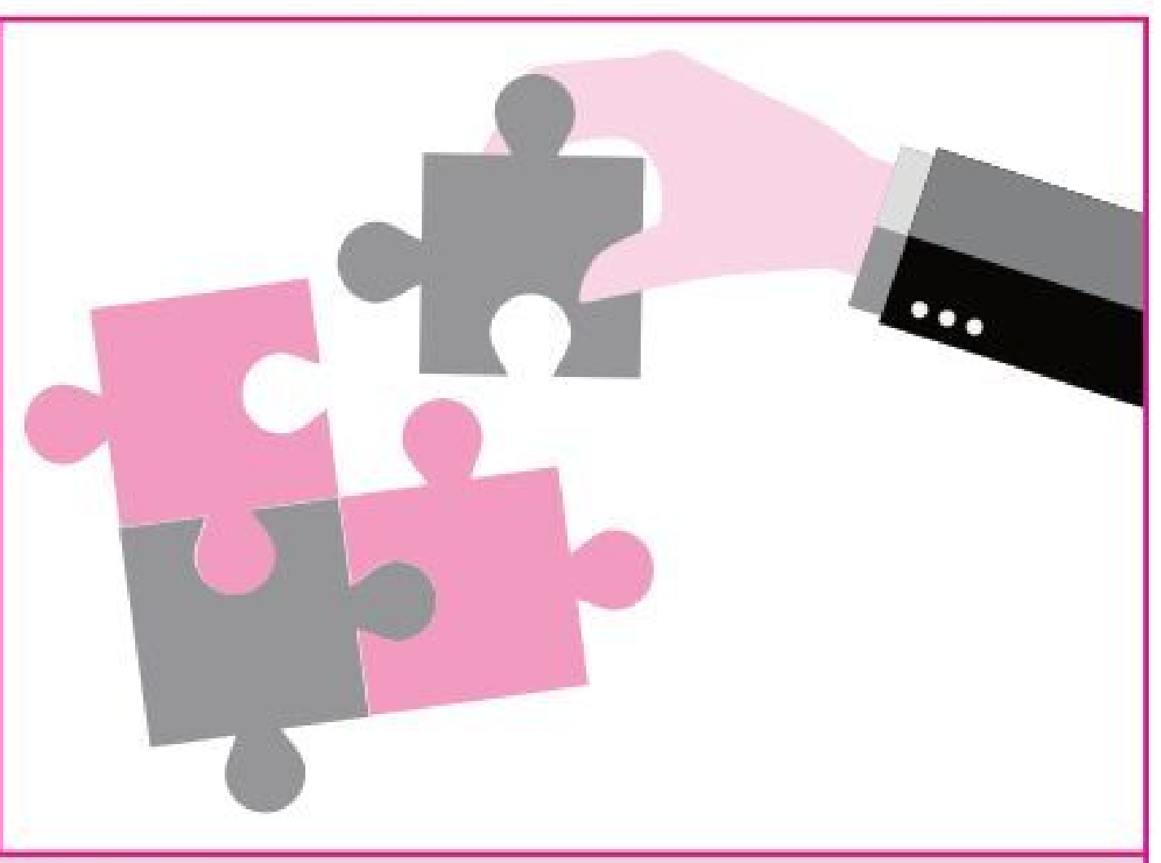
$= 0.26$

(b) P(both of them earn 10 points) $= P(A \cap B)$

$= P(A)\,P(B) = 0.8 \times 0.9 = 0.72$

◆ ◆

PUZZLE CORNER

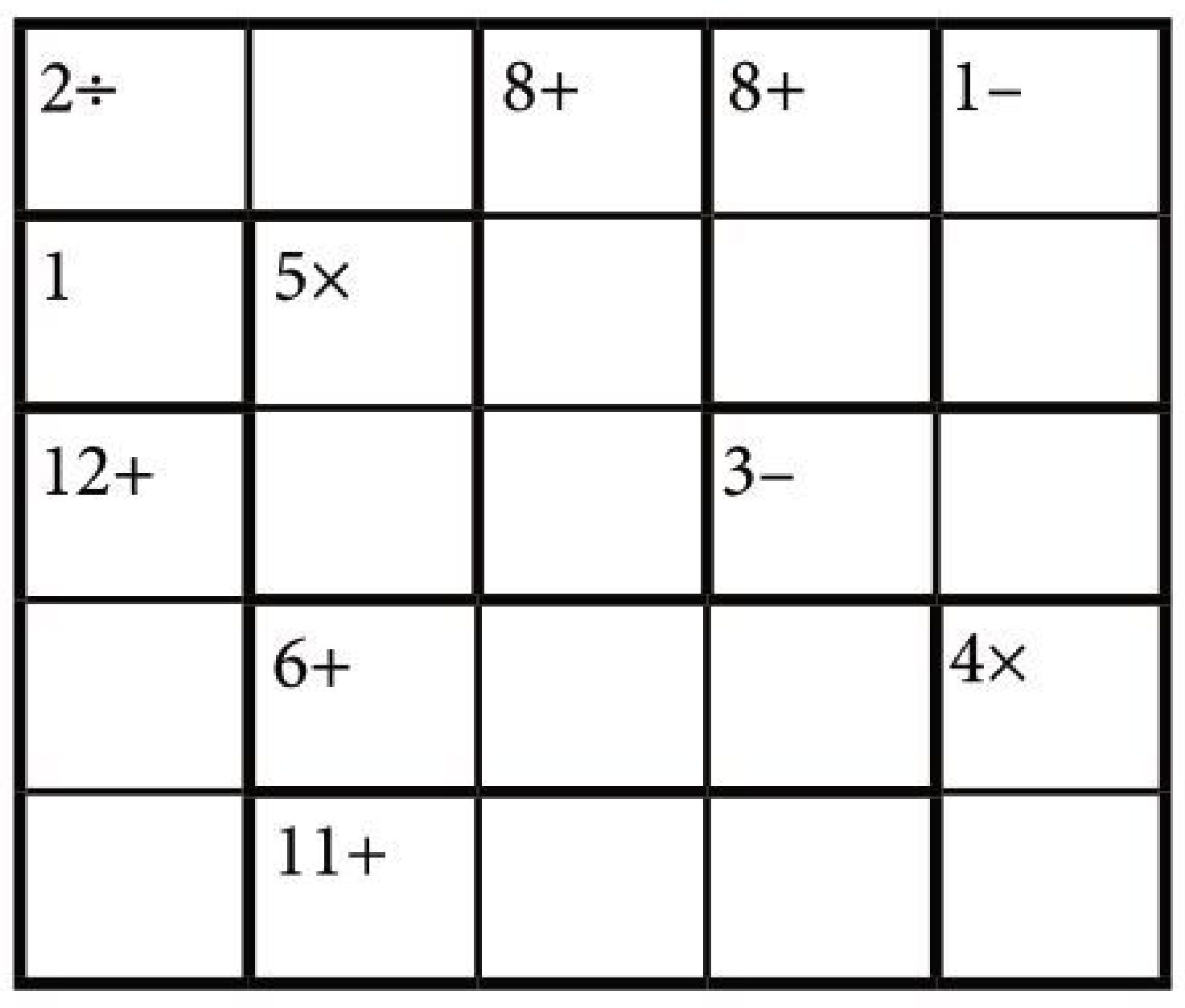


MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 5 × 5 grid is given, your objective is to fill the digits 1-5 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



2÷		8+	8+	1−
1	5×			
12+			3−	
	6+			4×
	11+			

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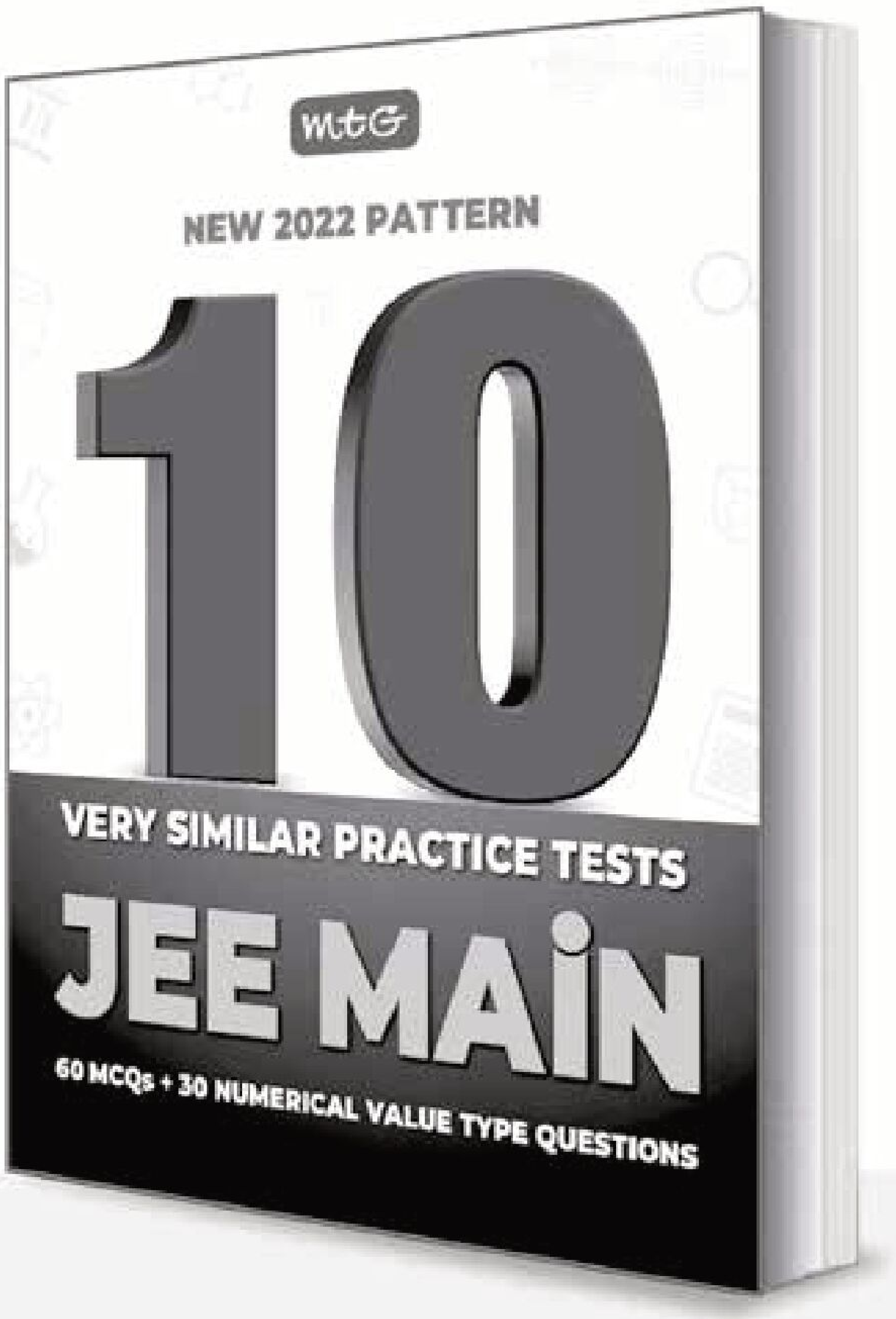
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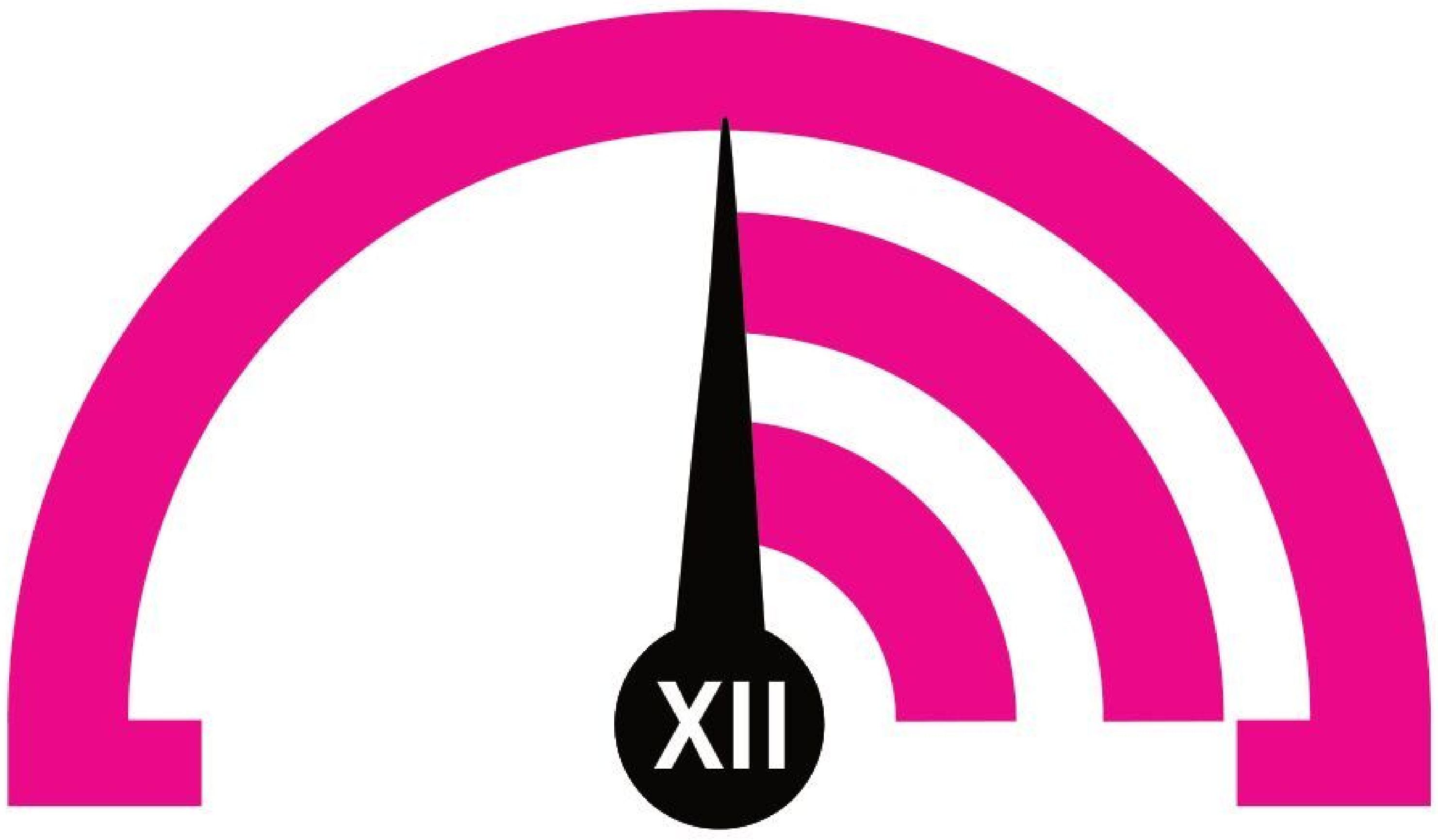

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. Consider the following relations :
$R = \{(x, y) : x, y$ are real and $x = wy$ for some non-zero rational number $w\}$
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) : m, n, p, q\right.$ are integers, such that $n, q \neq 0, mq = np\}$ then
(a) R is an equivalence relation, but S is not.
(b) S is an equivalence relation, but R is not.
(c) both R and S are equivalence relations.
(d) neither R nor S is an equivalence relation.

2. The points on the curve $y^3 + 3x^2 = 12y$ where tangent is vertical, are
(a) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (b) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$
(c) $(0, 0)$ (d) $\left(\pm\frac{11}{3}, 0\right)$

3. For the set of linear equations $\lambda x - 3y + z = 0$, $x + \lambda y + 3z = 1$, $3x + y + 5z = 2$ the value of λ, for which the equations does not have a unique solution.
(a) $-1, \frac{11}{5}$ (b) $-1, -\frac{11}{5}$
(c) $-\frac{11}{5}, 1$ (d) $1, \frac{11}{5}$

4. If from each of 3 boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn then the probability of drawing 2 white and 1 black ball is
(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{3}{16}$

5. $\sin^{-1}(\cos \sin^{-1} x) + \cos^{-1}(\sin \cos^{-1} x) =$
(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

6. If f is a continuous function and $\int_0^x f(t)dt \to \infty$ as as $|x| \to \infty$, then the number of points in which the line $y = mx$ intersects the curve $y^2 + \int_0^x f(t)dt = 2$ is
(a) 0 (b) 1
(c) atmost 1 (d) atleast 1

One or More Than One Option(s) Correct Type

7. Let $\int \frac{x^{1/2}}{\sqrt{(1-x^3)}} dx = \frac{2}{3} gof(x) + c$, then
(a) $f(x) = \sqrt{x}$ (b) $f(x) = x^{3/2}$
(c) $f(x) = x^{2/3}$ (d) $g(x) = \sin^{-1}x$

8. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q. If PQ is of constant length k. Such a curve passing through $(0, k)$ is
(a) a circle with centre $(0, 0)$
(b) $x^2 + y^2 = k^2$
(c) $(1 - k)x^2 + y^2 = k^2$
(d) $x^2 + (1 + k^2)y^2 = k^2$

9. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, III are p, q and 1/2 respectively. If the probability that the student is successful is 1/2, then
(a) $p = 1, q = 0$
(b) $p = 2/3, q = 1/2$
(c) $p = 3/5, q = 2/3$
(d) there are infinitely many values of p and q

10. The determinant

$$\Delta = \begin{vmatrix} b & c & b\lambda + c \\ c & d & c\lambda + d \\ b\lambda + c & c\lambda + d & a\lambda^3 + 3c\lambda \end{vmatrix}$$ is equal to zero, if

(a) b, c, d are in AP (b) b, c, d are in GP
(c) b, c, d are in HP
(d) λ is a root of $ax^3 - bx^2 + cx - d = 0$

11. If $P(2, 3, 1)$ is a point and $L \equiv x - y - z - 2 = 0$ is a plane then
(a) origin and P lie on the same side of the plane.
(b) distance of P from the plane is $\frac{4}{\sqrt{3}}$.
(c) foot of perpendicular is $\left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$.
(d) image of point P by the plane is $\left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$.

12. If $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0$, $a > 0$, $0 \le x \le 2a$, then
(a) $\phi(x)$ increases in $(a, 2a)$
(b) $\phi(x)$ increases in $(0, a)$
(c) $\phi(x)$ decreases in $(a, 2a)$
(d) $\phi(x)$ decreases in $(0, a)$

13. Which of the following functions are periodic?
(a) $f(x) = \sin x + |\sin x|$
(b) $g(x) = \frac{(1 + \sin x)(1 + \sec x)}{(1 + \cos x)(1 + \operatorname{cosec} x)}$
(c) $h(x) = \max(\sin x, \cos x)$
(d) $p(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3x + 10$, where [.] denotes the greatest integer function.

Comprehension Type

Paragraph for Q. No. 14 to 15

Let A be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

14. The number of matrices in A is
(a) 12 (b) 6 (c) 9 (d) 3

15. The number of matrices A in A for which the system of equations $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is
(a) 0 (b) more than 2
(c) 2 (d) 1

Matrix Match Type

16. Match the following.

	Column-I		Column-II
(P)	A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$, $\frac{x-8/3}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. PQ^2 is	(1)	7
(Q)	The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\frac{3}{5}$ are	(2)	6
(R)	Non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a} \cdot \vec{b} = 0$, $(\vec{b} - \vec{a}) \cdot (\vec{b} + \vec{c}) = 0$, $2\vert\vec{b} + \vec{c}\vert = \vert\vec{b} - \vec{a}\vert$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the values of μ are	(3)	5
(S)	Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \frac{\sin 9x/2}{\sin x/2}$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(4)	4

	P	Q	R	S		P	Q	R	S
(a)	2	4	3	4	(b)	3	4	2	1
(c)	3	2	1	4	(d)	1	2	3	4

Numerical Answer Type

17. The distance between the skew lines $\frac{x}{4} = \frac{y-7}{-3} = \frac{z-7}{-2}$ and $\frac{x-4}{2} = \frac{y+6}{3} = \frac{z-34}{-10}$ is

18. The resultant of vectors $\vec{a}$ and $\vec{b}$ is $\vec{c}$. If $\vec{c}$ trisects the angle between $\vec{a}$ and $\vec{b}$ and $|\vec{a}| = 6, |\vec{b}| = 4$, then $|\vec{c}|$ is

19. Let $A = \{1, 2, 3, 4\}$ and $B = \{0, 1, 2, 3, 4, 5\}$. If m is the number of increasing functions from A to B and n is the number of onto functions from B to A, then the last digit of $n - m$ is

20. The area bounded by the y-axis, the tangent and normal to the curve $x + y = x^y$ at the point where the curve cuts the x-axis is

◆◆

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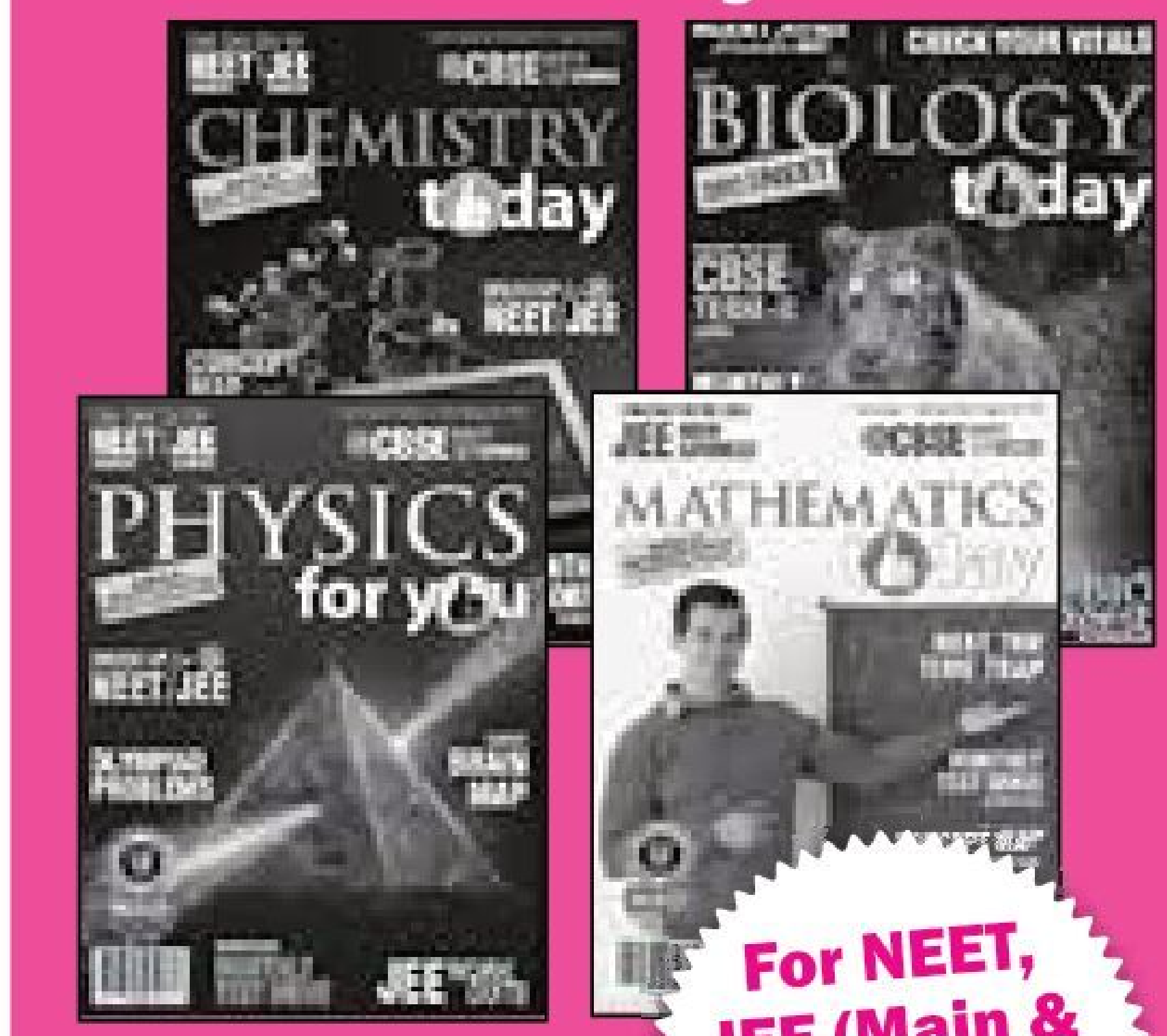

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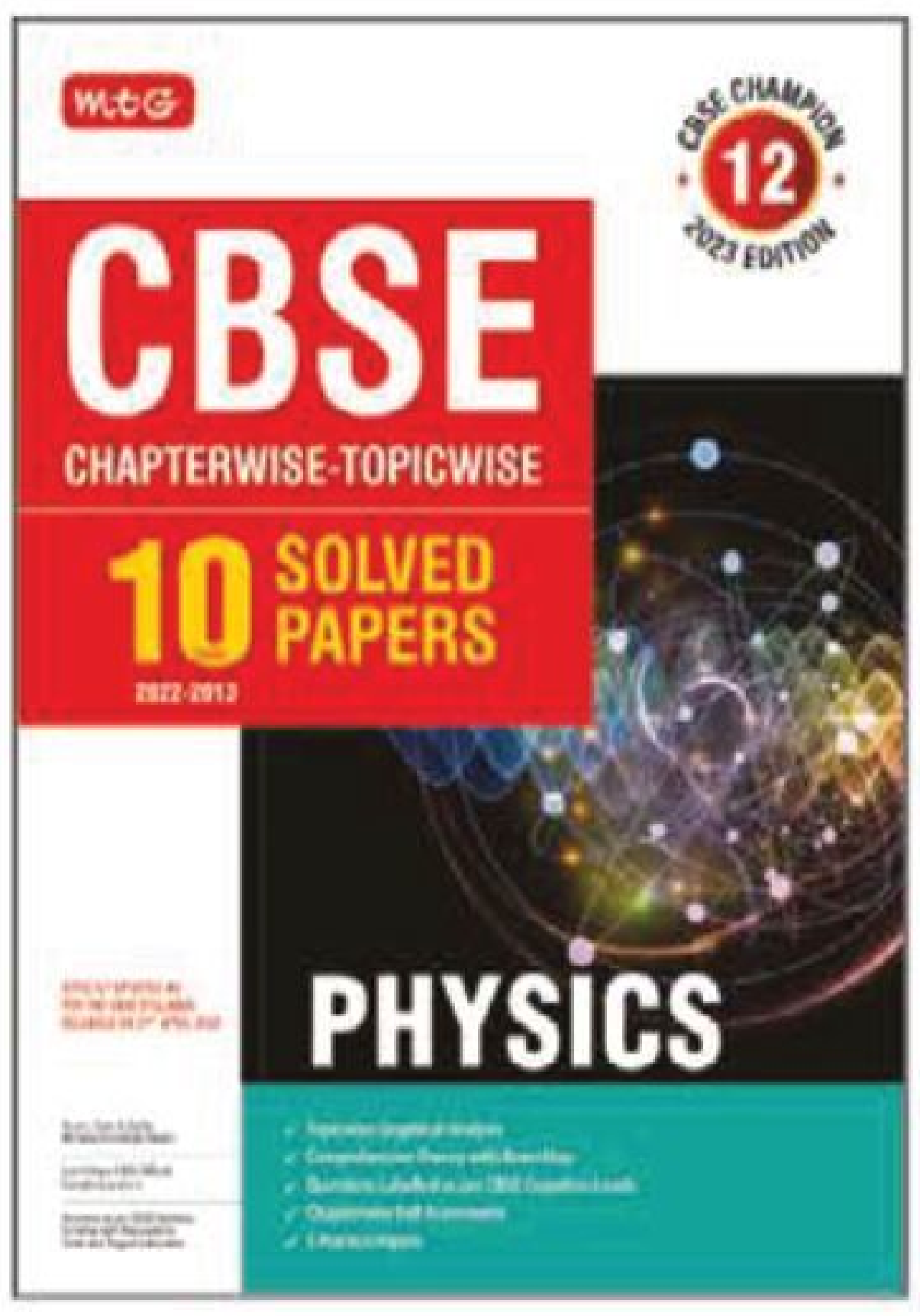
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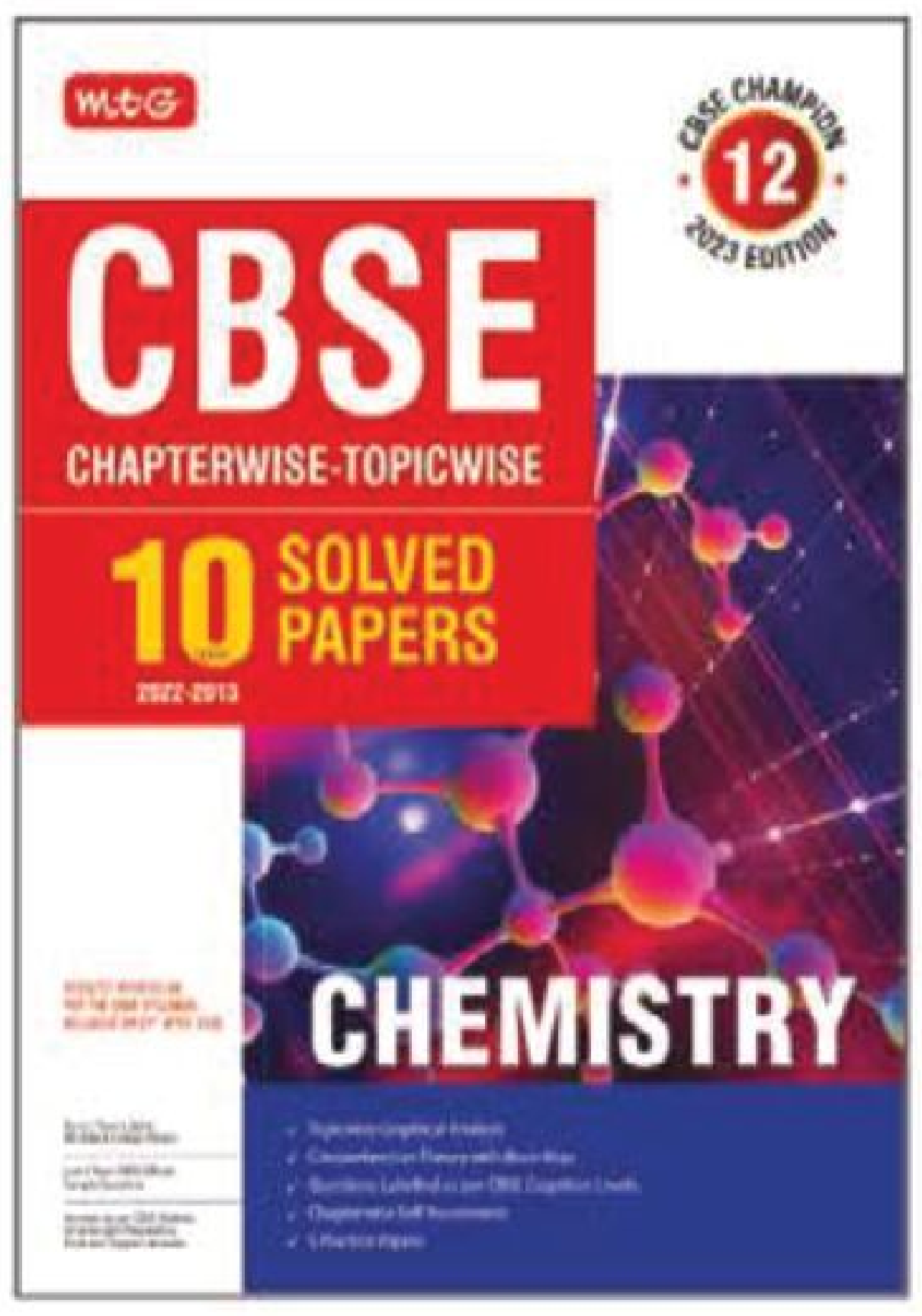
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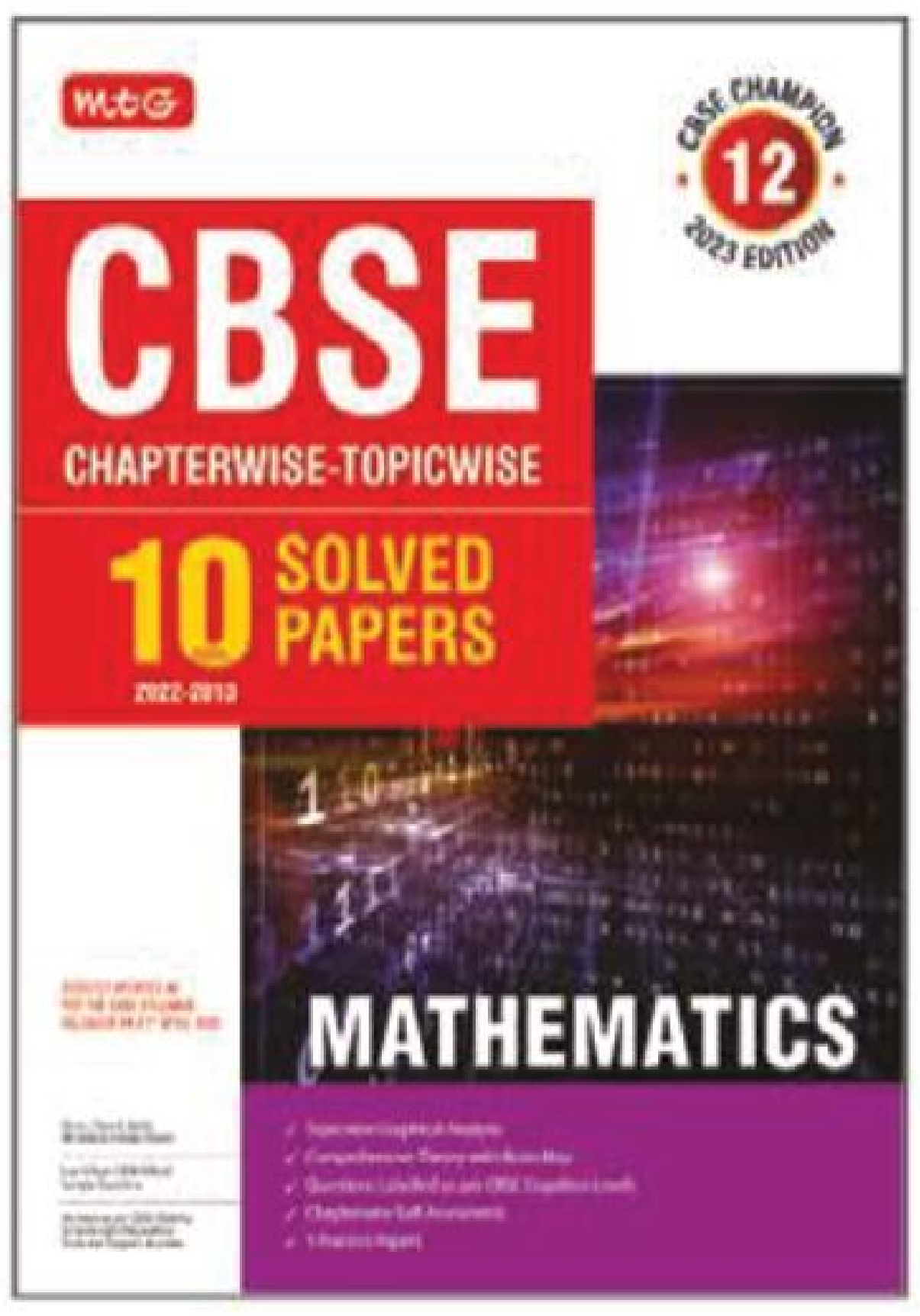
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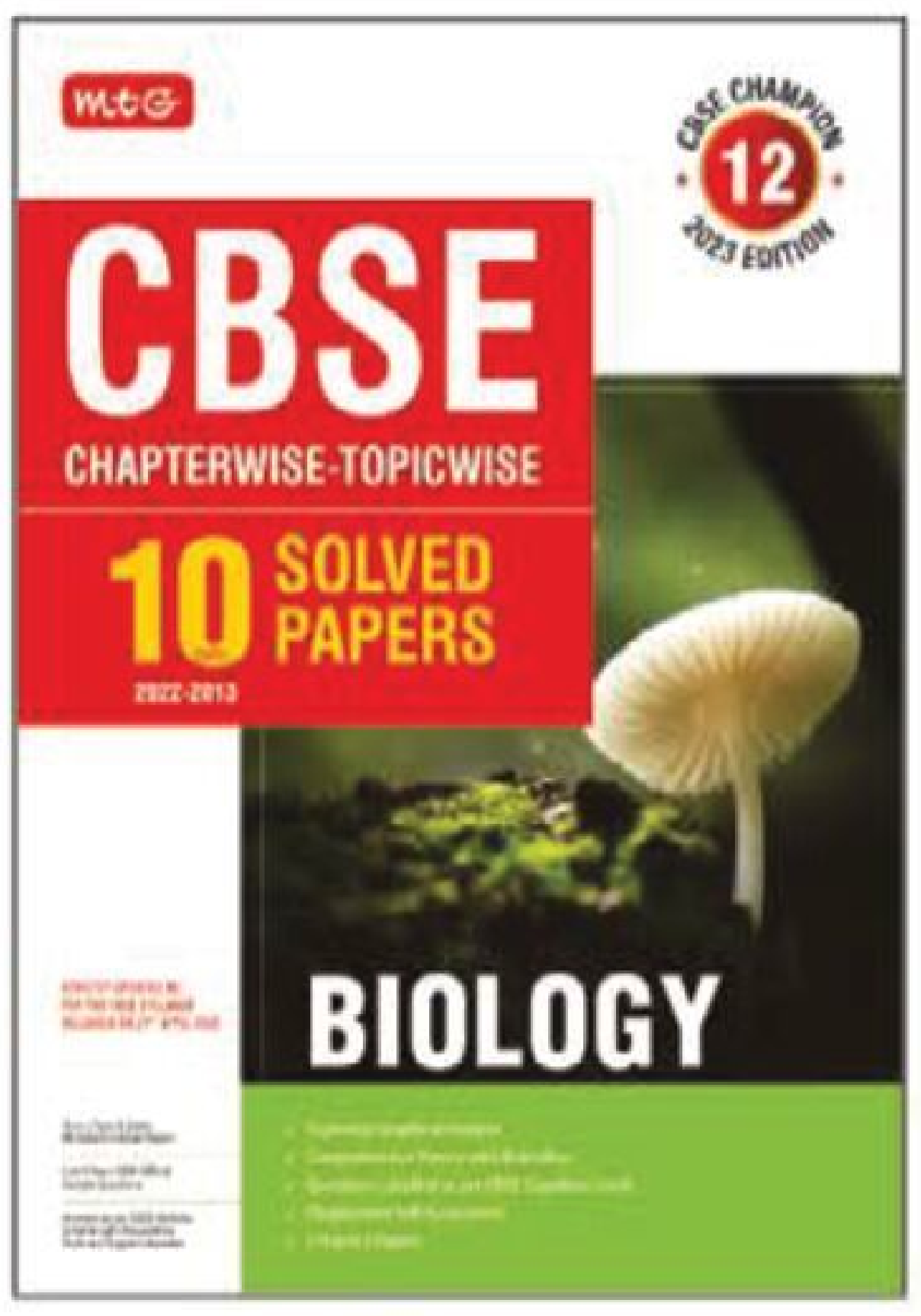
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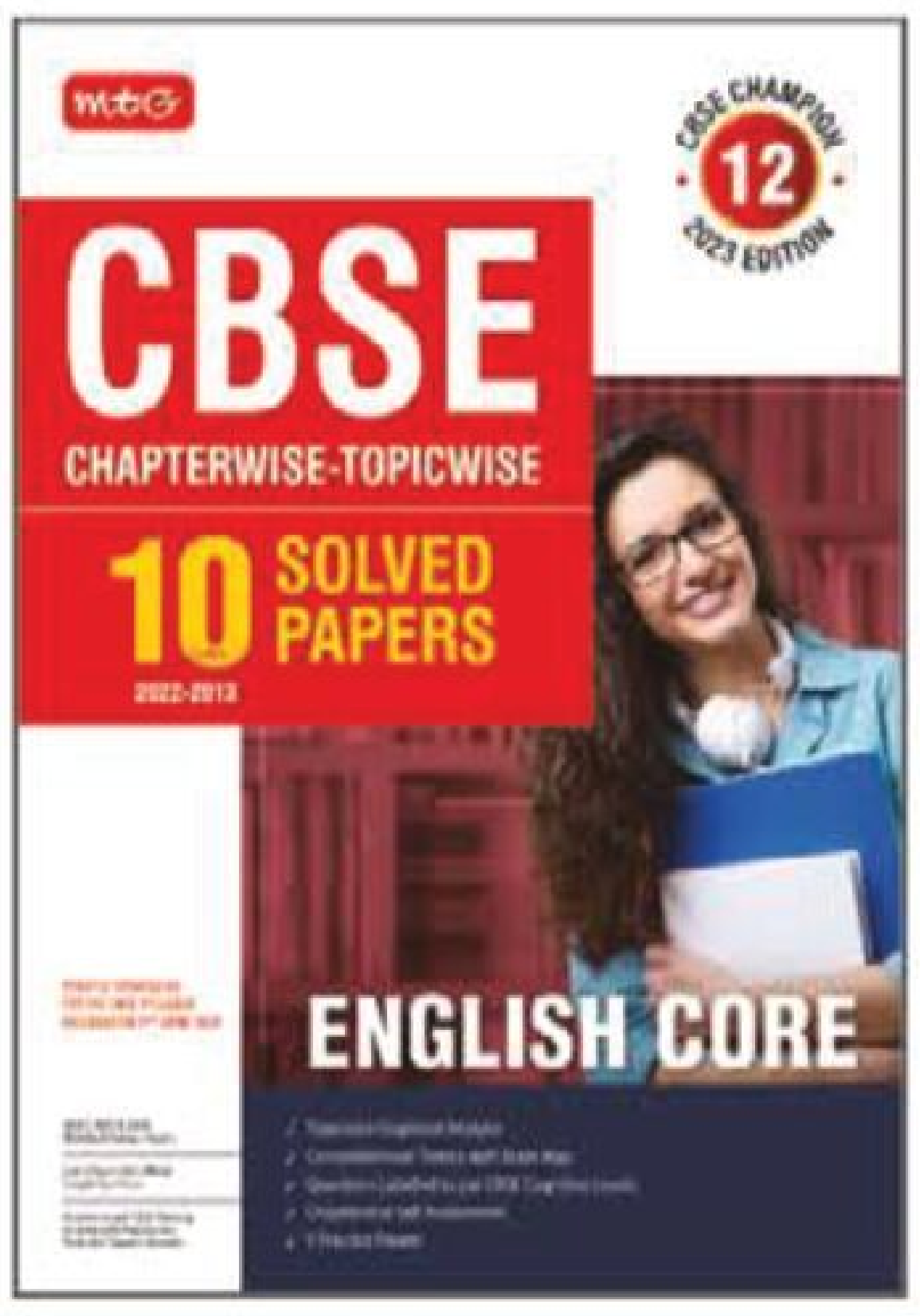
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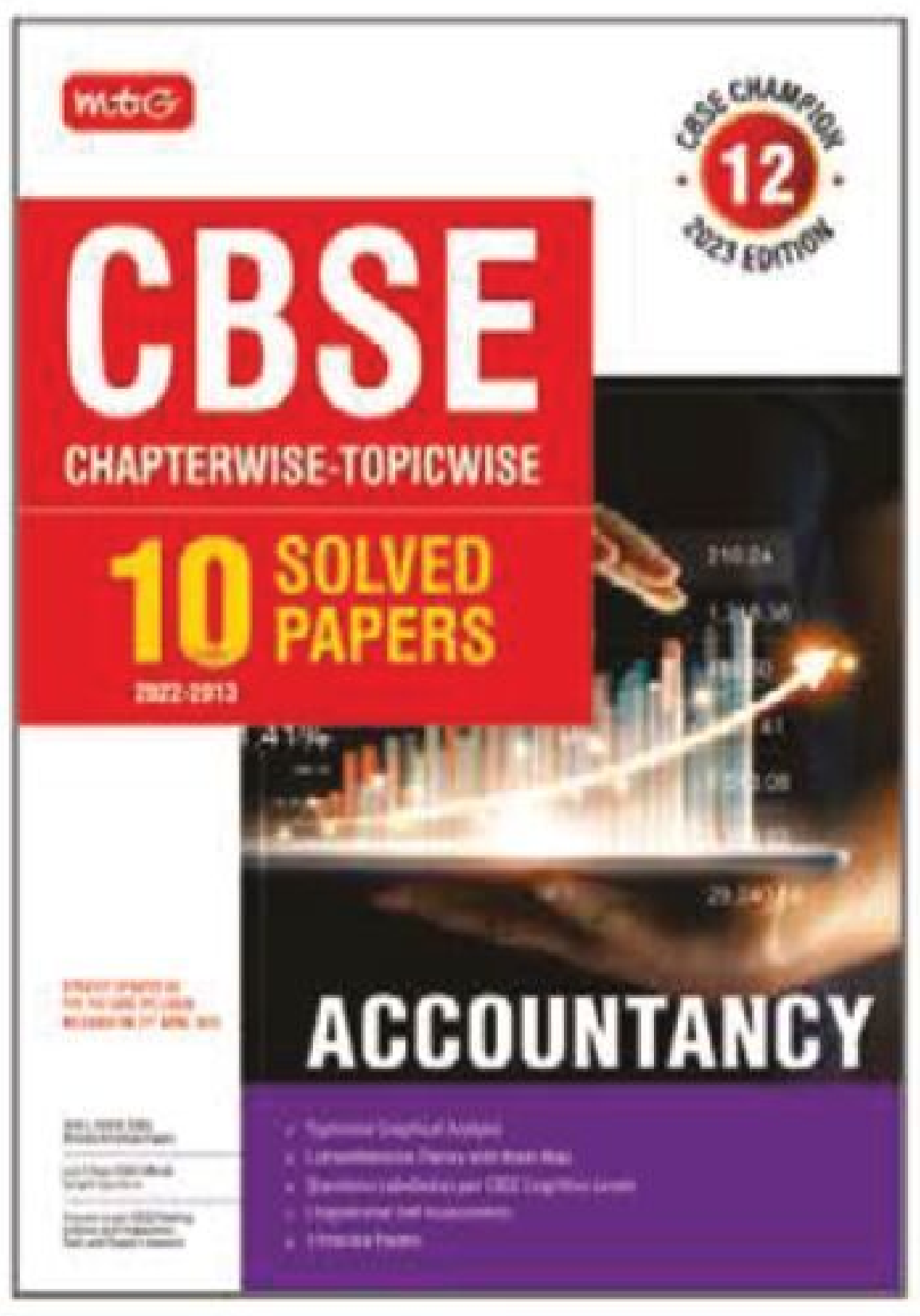
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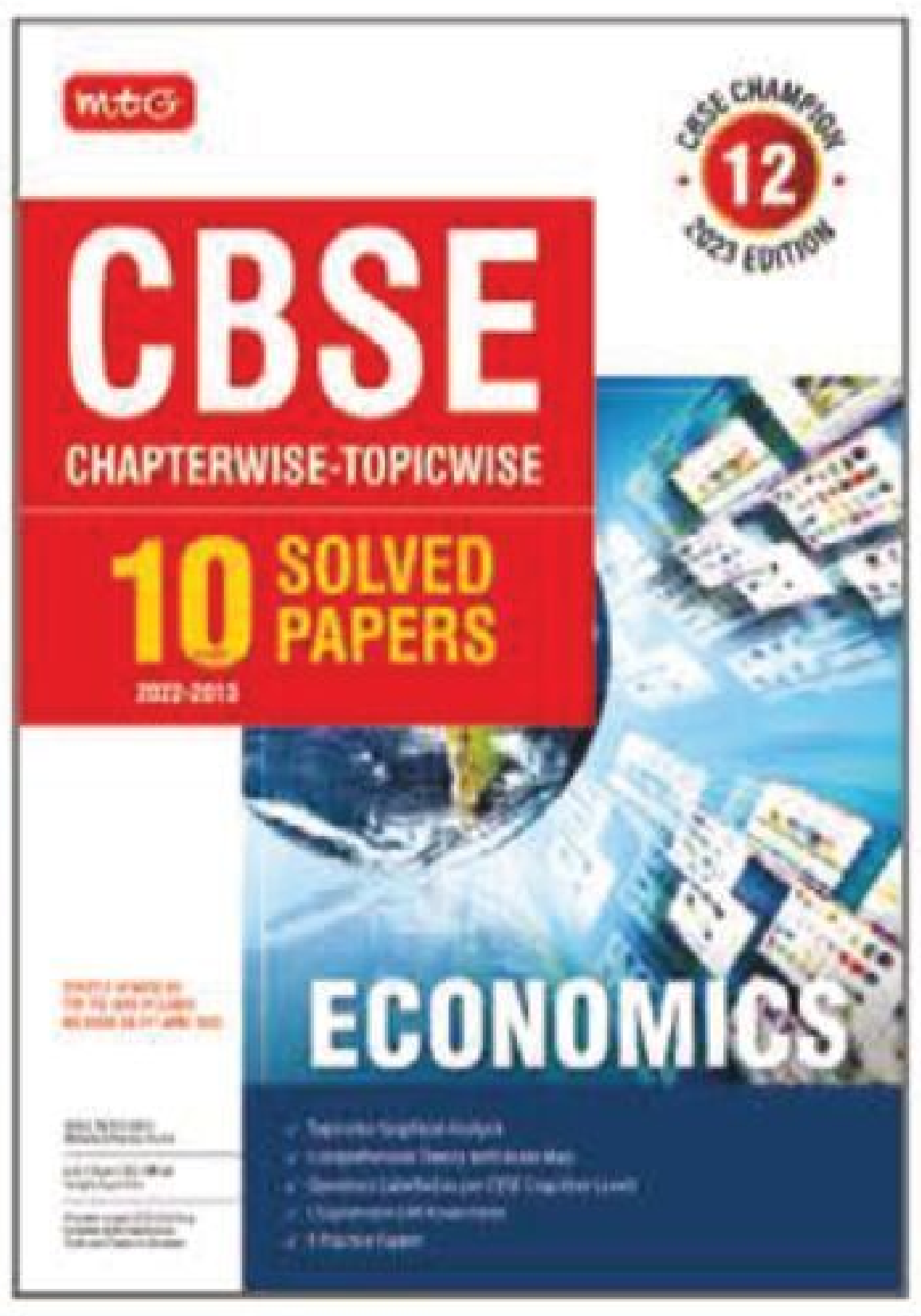
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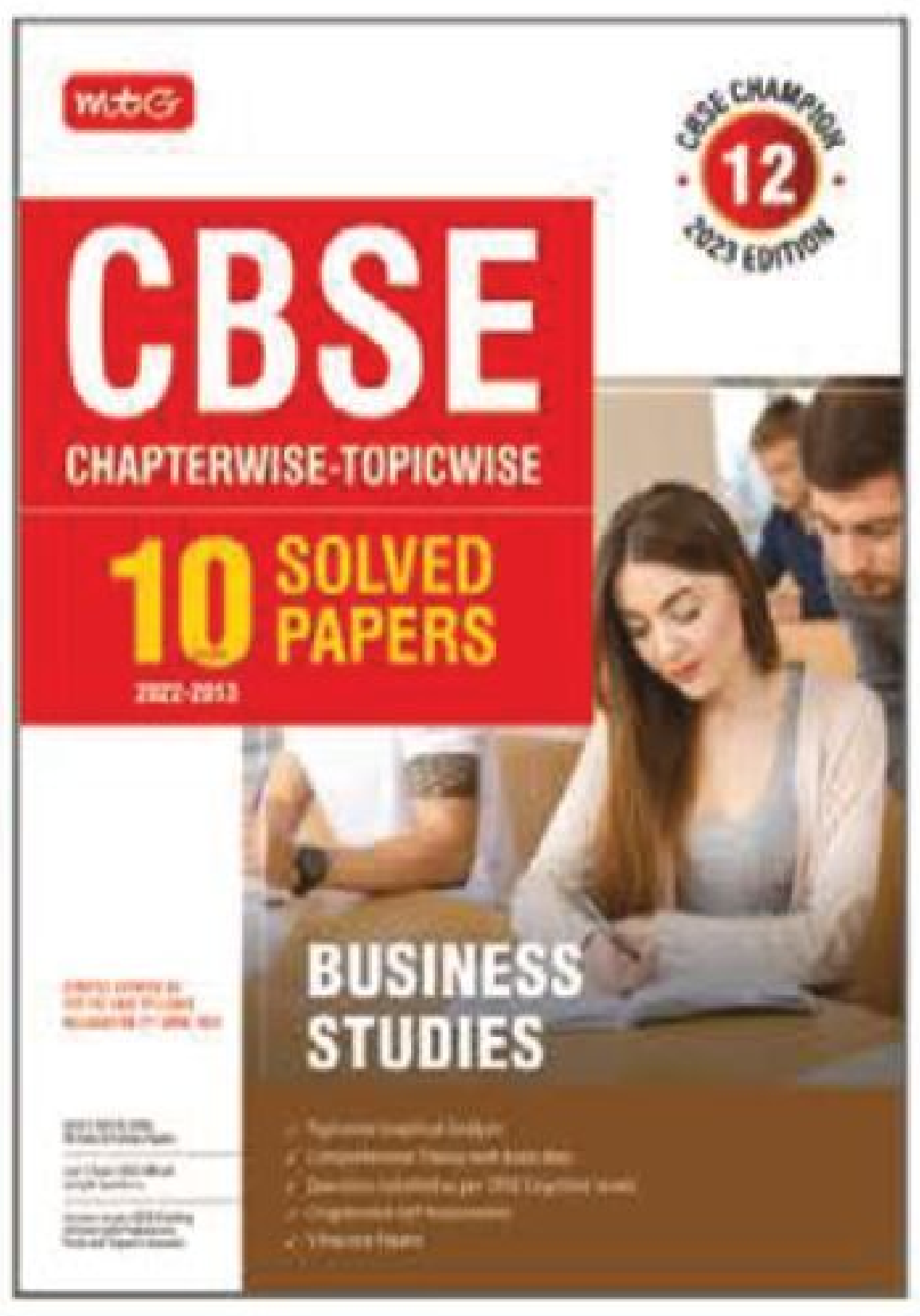
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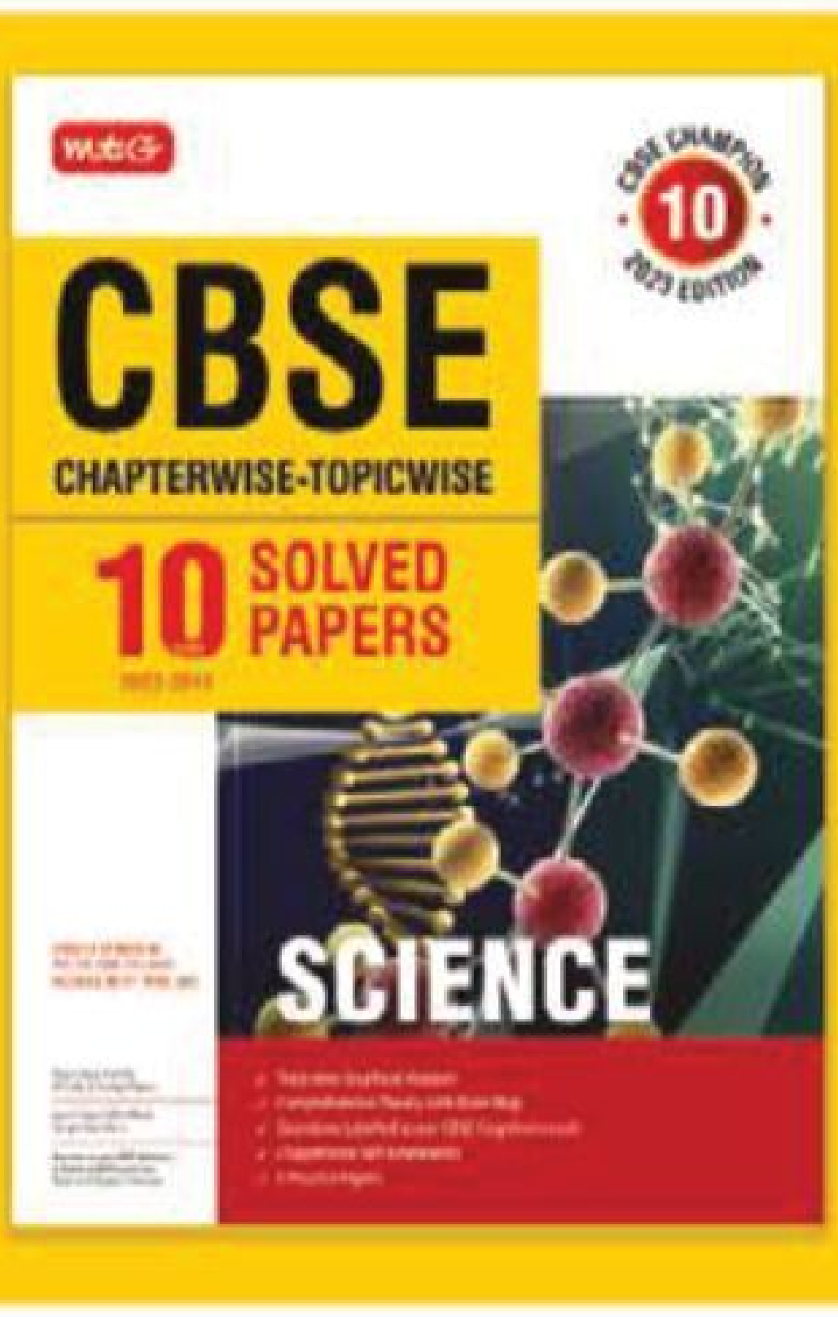
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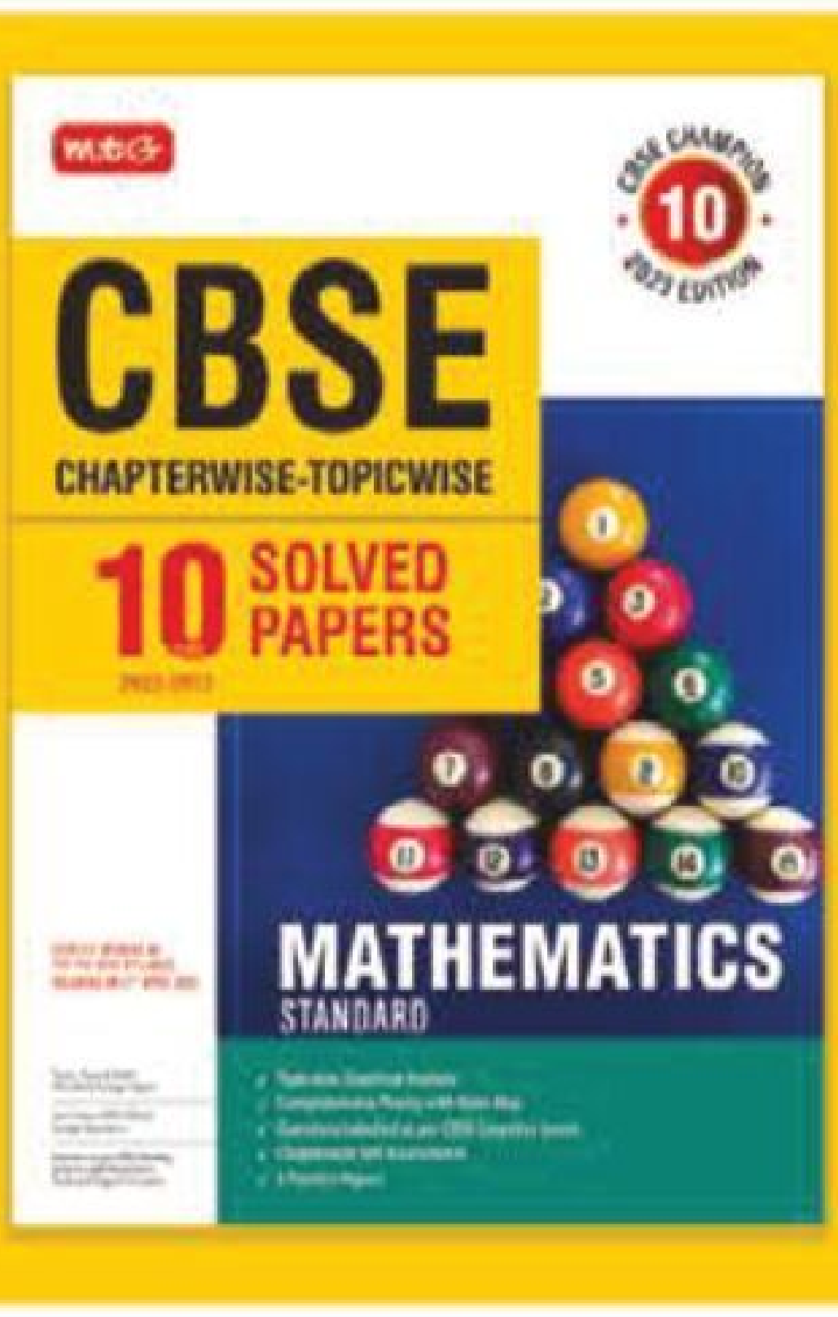
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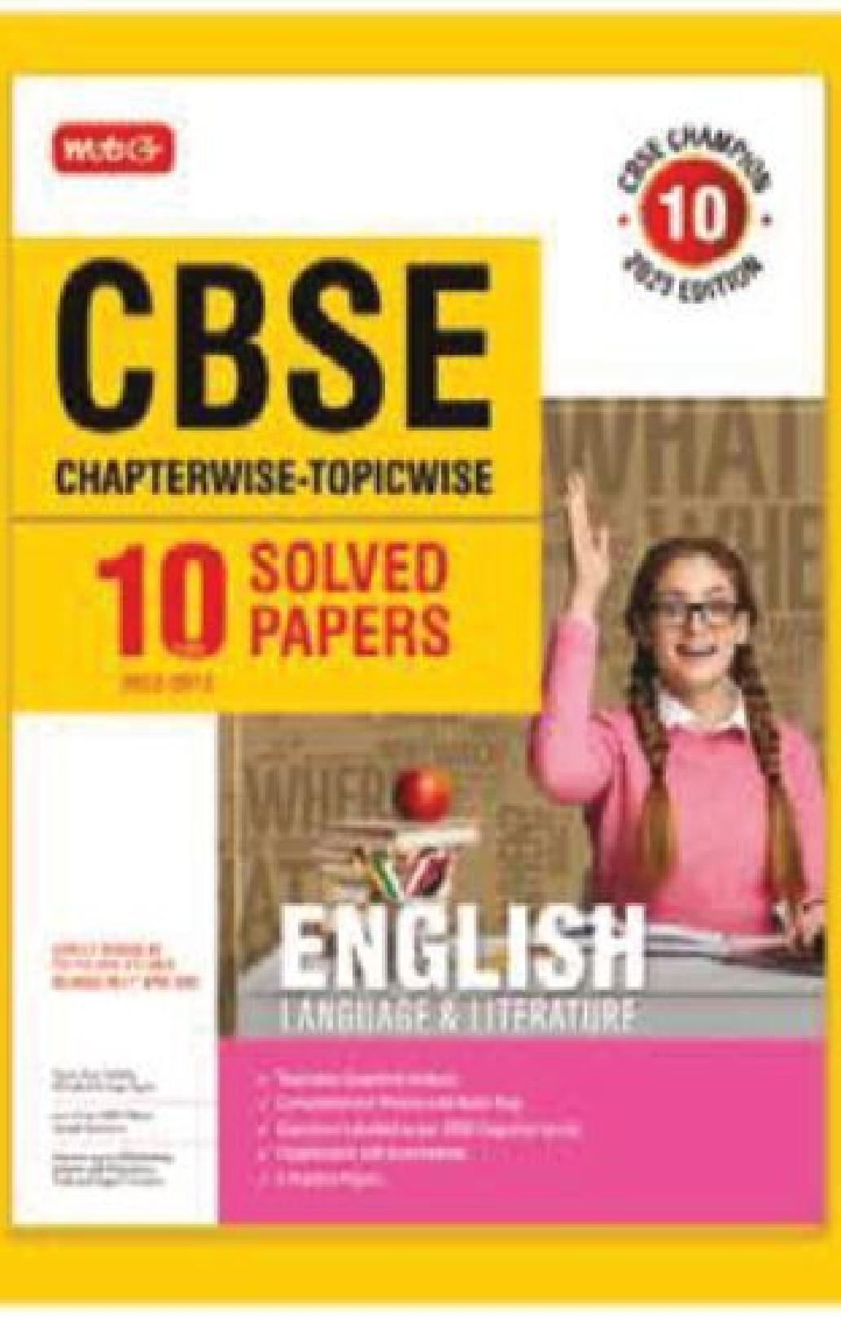
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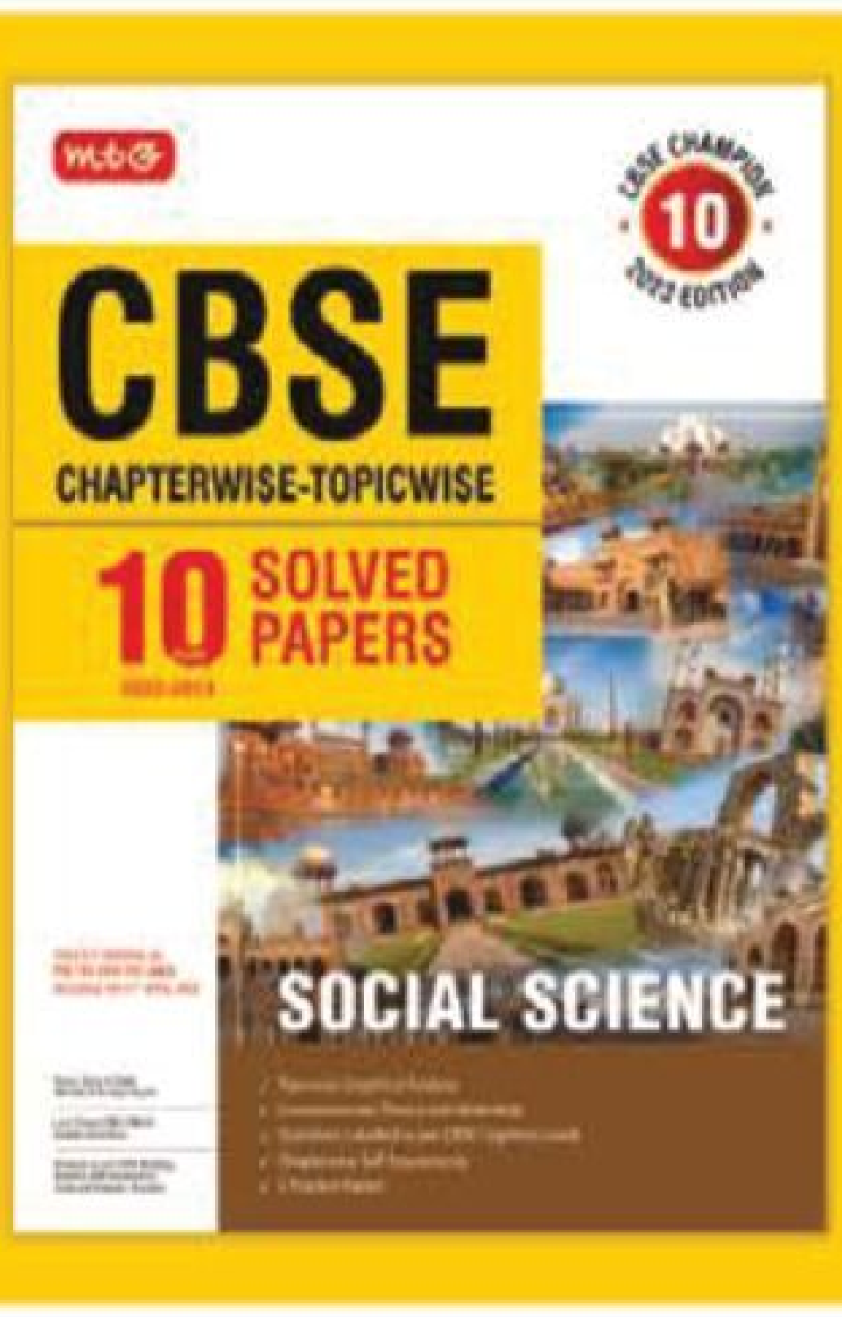
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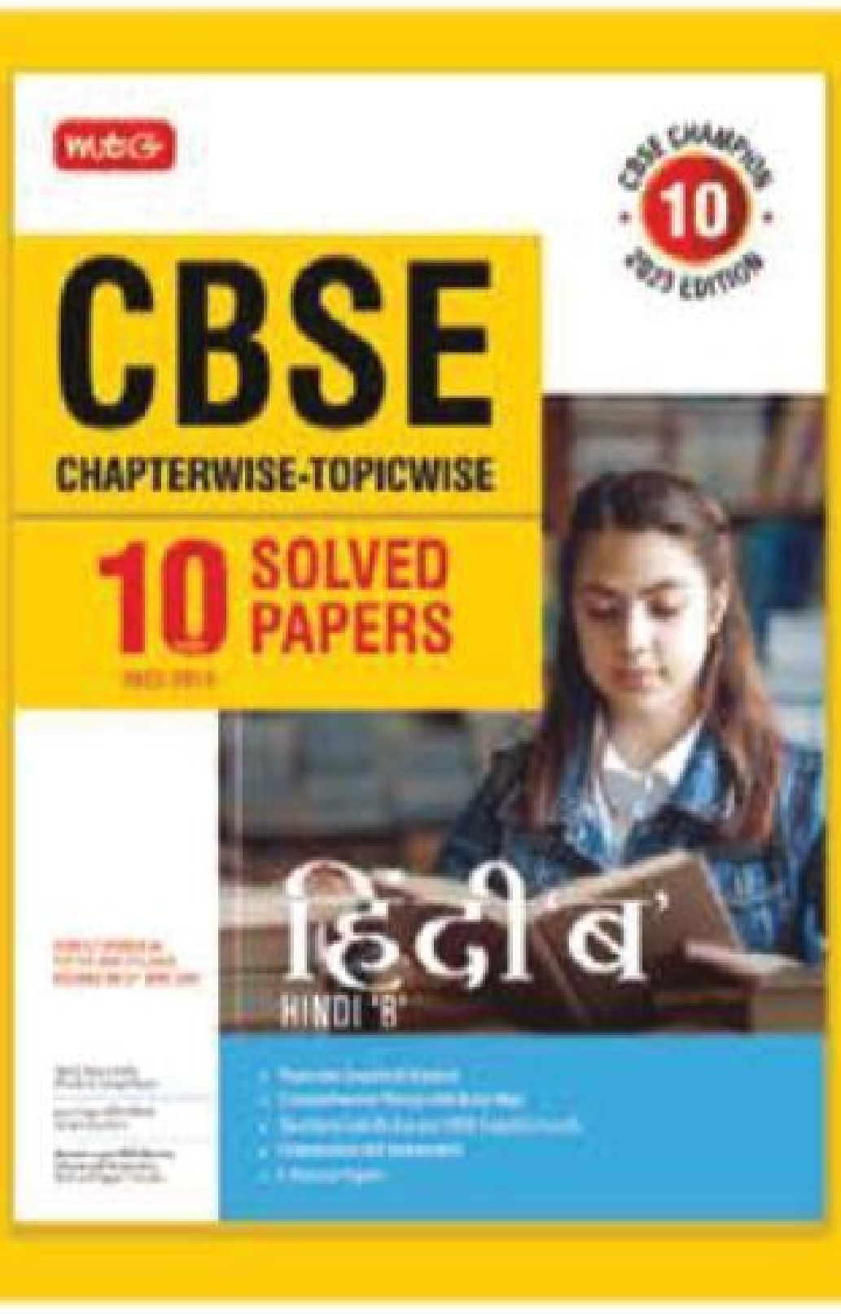
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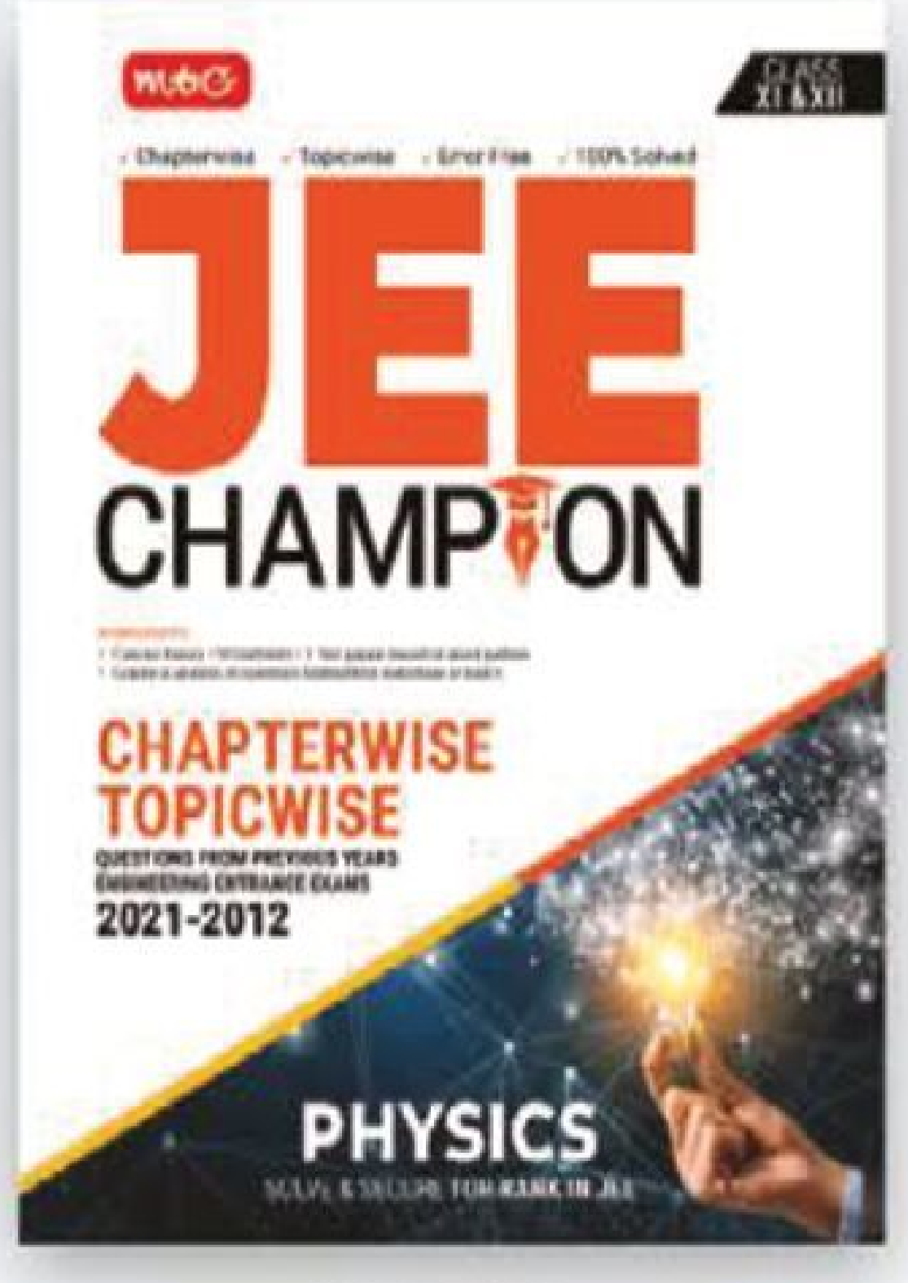
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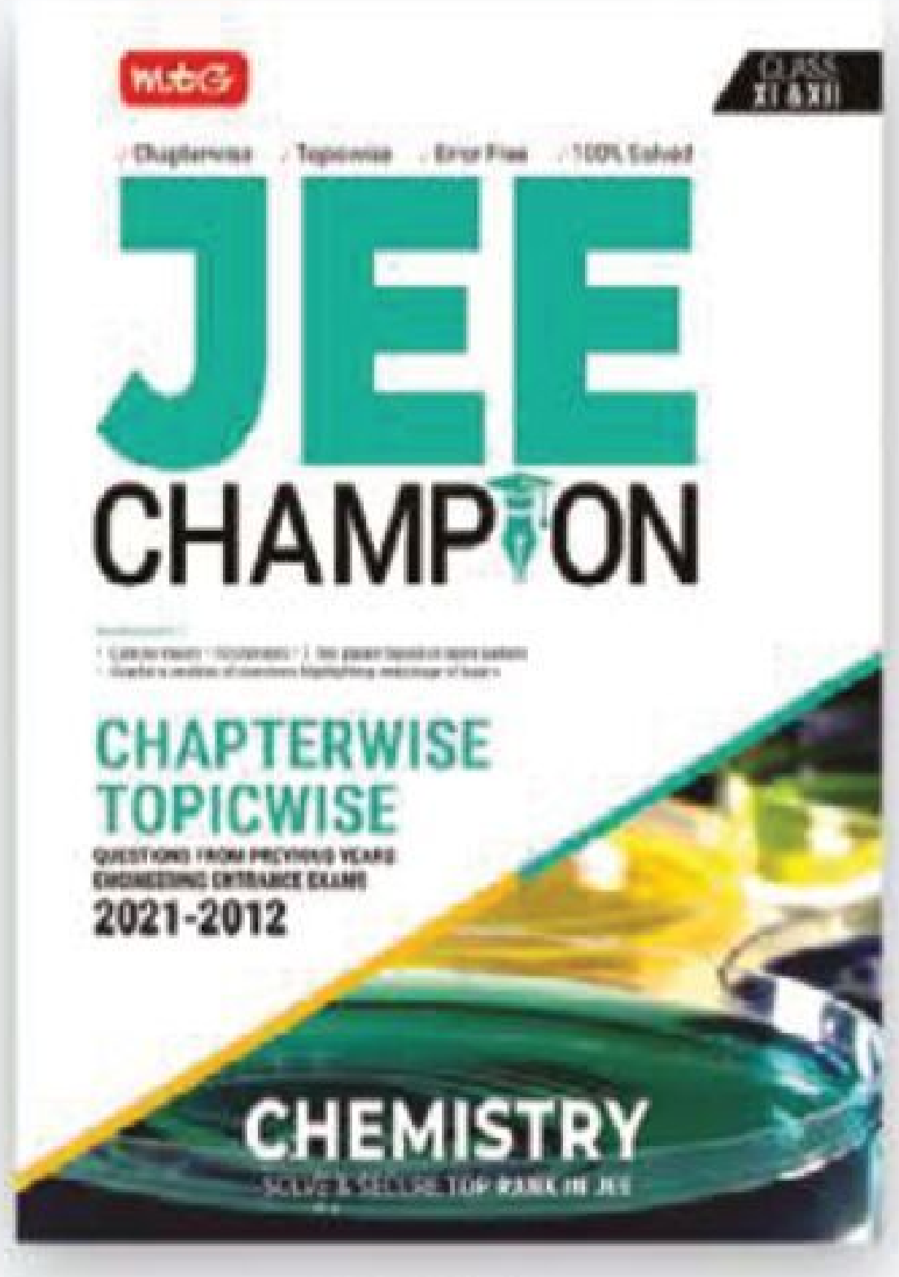
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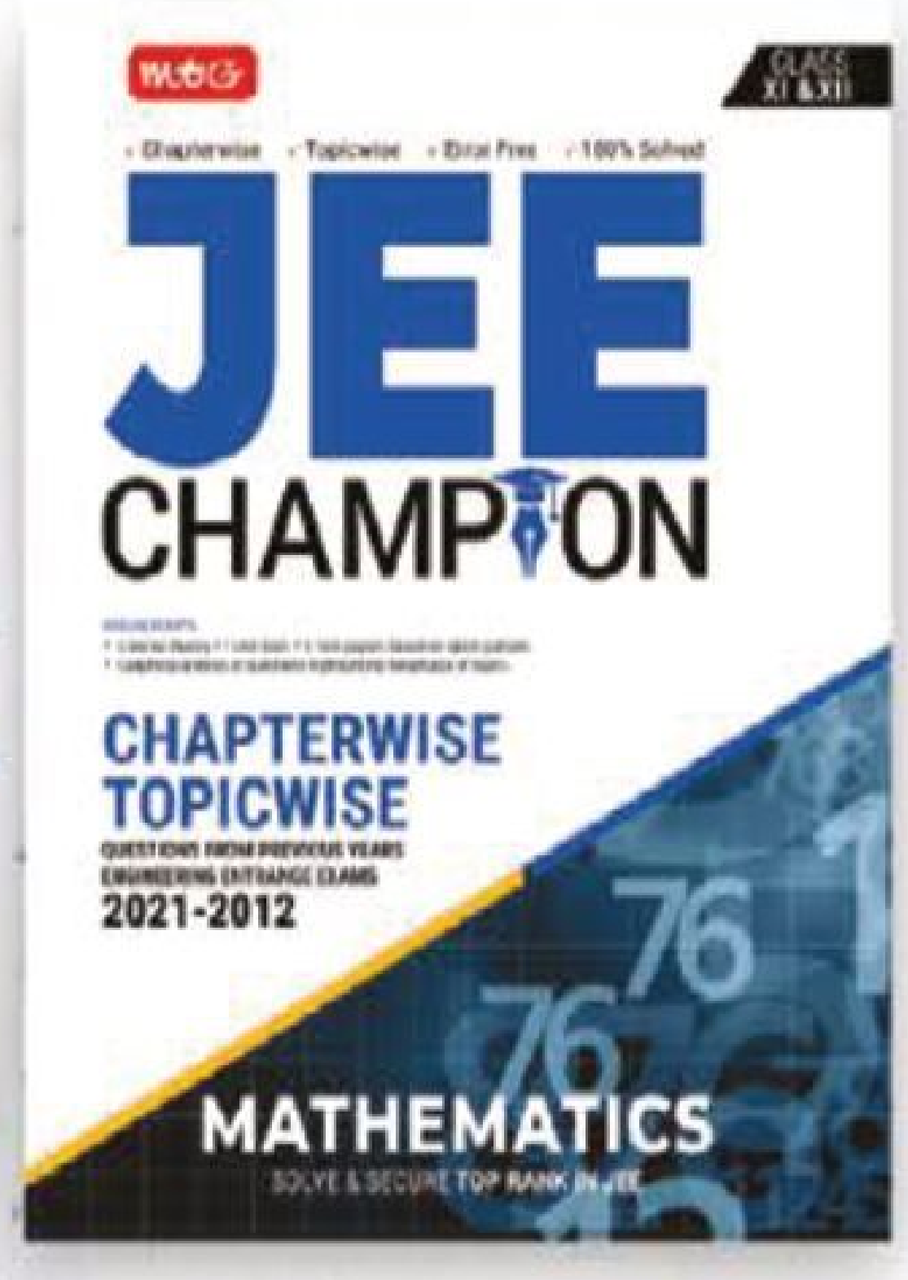
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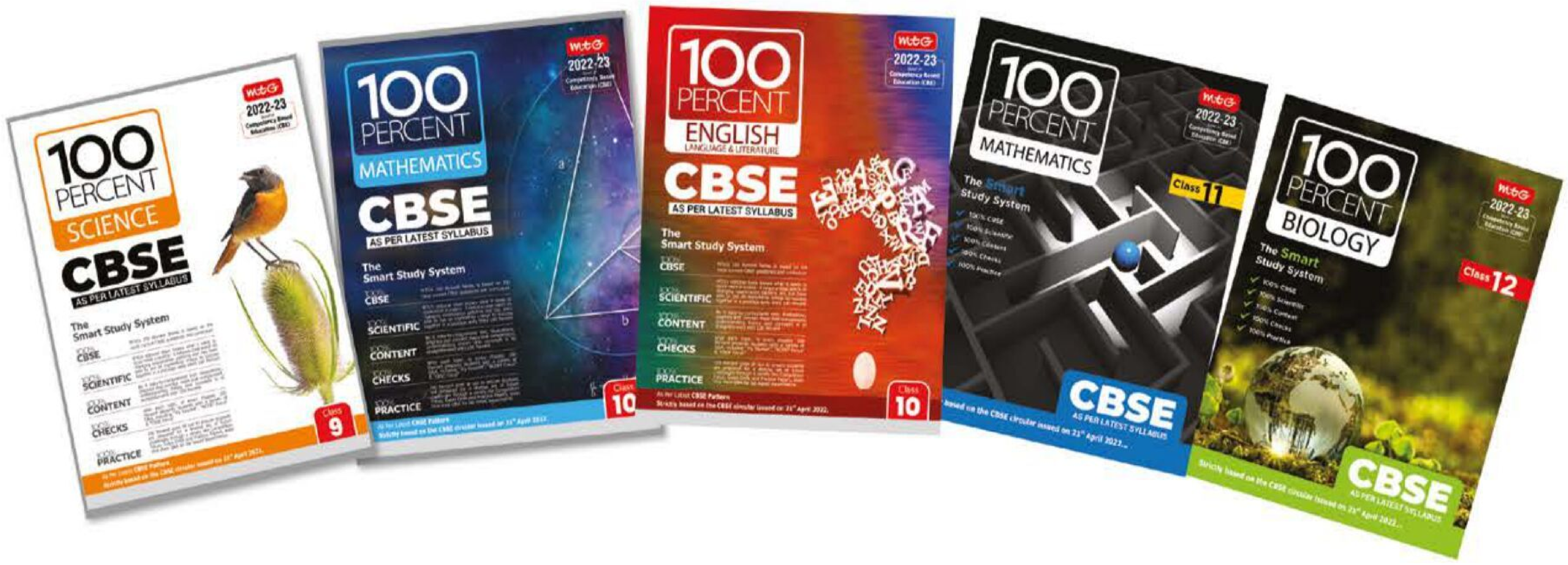

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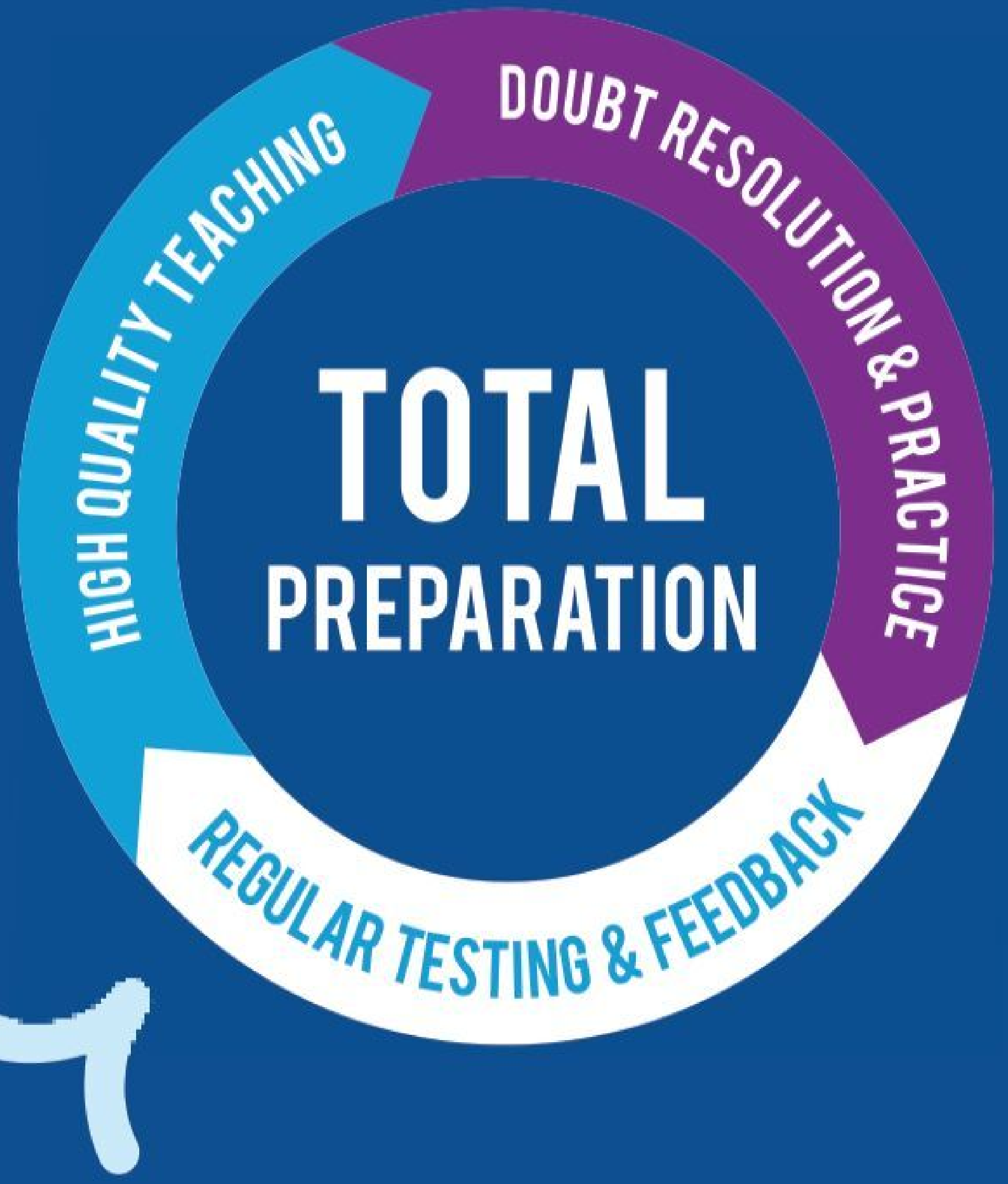
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NEET-UG

Hat-trick of Perfect Scores

AIR 1
Perfect Score 720/720
AIR 1
Perfect Score 720/720
AIR 1
Perfect Score 720/720

MRINAL KUTTERI 2 Year Classroom
KARTHIKA G NAIR 2 Year Classroom
TANMAY GUPTA 2 Year Classroom

JEE (Main)

Our Top Performers

AIR 3
AIR 9
AIR 25

PRAVAR KATARIA 5 Year Classroom
ANMOL ARICHWAL 4 Year Classroom
RANJIM PRABAL DAS 5 Year Classroom
and many more...

JEE (Advanced)

Our Top Performers

AIR 58
AIR 60
AIR 75

ANMOL ARICHWAL 4 Year Classroom
PRAVAR KATARIA 5 Year Classroom
VINAY TAYAL 2 Year Classroom
and many more...

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